

KIPS
ENTRY TESTS
SERIES

PREP BOOK



PHYSICS

National MDCAT

AS PER PMC SYLLABUS

- ▶ Topic-wise Complete Syllabus
- ▶ Comprehensive Course Revision
- ▶ Detailed Explanation of Topics
- ▶ Tables, Flow Sheets & Diagrams
- ▶ Critical Concepts; Critical Thinking



A Kitab Dost Publication

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TOPIC-1 >> FORCE AND MOTION

COURSE CONTENT

- Displacement
- Velocity
- Displacement-time graph
- Acceleration
- Uniform acceleration
- Variable acceleration
- Graphical representation of acceleration with velocity time graph
- Newton's laws of motion (1st 2nd, and 3rd law)
- Linear Momentum
- Law of conservation of momentum
- Collision
- Elastic collision
- Elastic collision in one dimension
- Elastic collision in one dimension under different cases
- Projectile motion
- Characteristics of projectile motion
- Time of flight
- Maximum height
- Horizontal range

DISPLACEMENT, VELOCITY

Rest and Motion

The concept of state of motion and rest are described relative to the observer.

- If a body does not change its state w.r.t surroundings, it is in the state of rest.
- If a body changes its state w.r.t surroundings, then it is in the state of motion.
- A moving body can possess both states of rest and motion, depending on the observer.

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g. Motion of car on a straight road. Motion of freely falling body.	e.g. Motion of car on a circular turn. Motion of billiards ball.	e.g. Motion of flying kite. Motion of flying insect.

Distance

The total actual path traversed by a body between its initial and final positions is called distance. It is a scalar quantity.

Displacement

The change in the position of a body in a certain direction from initial position to the final position is known as displacement. It is a vector quantity.

- Displacement may be positive, negative or zero but distance is always positive.

Speed

It is defined as distance traveled in one second.

Velocity

Time rate of change of displacement is called velocity. It is a vector quantity and may be positive or negative.

$$\bar{v} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta \bar{d}}{\Delta t}$$

$$\bar{v} = \frac{\bar{d}_2 - \bar{d}_1}{t_2 - t_1}$$

Distance covered in nth second
 $S_n = u_0 + \frac{a}{2}(2n+1)$ n = second
Body moving with uniform after 8 sec $a = 25 \text{ ms}^{-1}$
 after 8 sec $= 30 \text{ ms}^{-1}$ - *Distance covered in 10*

Uniform Velocity

When a body covers equal displacements in equal interval of time, however small this time interval may be, then its velocity is said to be uniform.

- If a body travels with uniform velocity \bar{v}_1 for time t_1 and with uniform velocity \bar{v}_2 for time t_2 , then its average velocity will be

diff. distance
diff. Speed

$$\bar{v}_{\text{avg}} = \frac{S_1 + S_2}{\frac{S_1}{\bar{v}_1} + \frac{S_2}{\bar{v}_2}}$$

Example

A car traveled the first third of distance S at speed of 10 m s^{-1} , the second third at a speed of 20 m s^{-1} and last third at a speed of 60 m s^{-1} . Determine average speed of car.

$\begin{array}{c} S/3 \quad S/3 \quad S/3 \\ | \quad | \quad | \\ \text{A} \quad 10 \quad \text{B} \quad 20 \quad \text{C} \quad 60 \quad \text{D} \end{array}$

$$v_m = \frac{\text{Total distance traveled}}{\text{total time}} = \frac{AB + BC + CD}{t_1 + t_2 + t_3} = \frac{S/3 + S/3 + S/3}{\frac{S/3}{10} + \frac{S/3}{20} + \frac{S/3}{60}} = 18 \text{ ms}^{-1}$$

Instantaneous Velocity

The instantaneous velocity \vec{v} of a body is defined as the

limit of the ratio of change in position $\Delta \vec{d}$ (displacement) to the small-time interval Δt as Δt following an instant 't' approach zero:

$$\vec{v}_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{d}}{\Delta t}$$

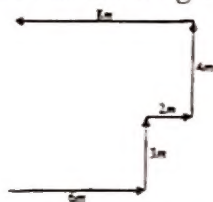
For your information

- The magnitude of velocity is called speed
- Velocity = Speed \times direction
- Speed and velocity, both have same unit ms^{-1}
- Speed is a scalar quantity whereas velocity is a vector quantity.

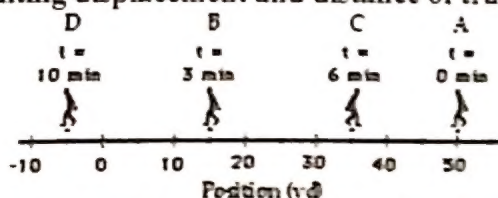
Typical Speeds	
Motion	Speed ms^{-1}
Walking Ant	0.01
Human Swimming	2
Human Running	4
Flying Bee	5
Tortoise	9
100 Meters Dash	10
Running Cheetah	29
Falcon in a dive	37
Automobile	62
Jet Airline	267
Sound in Air	333
Moon around the earth	1023
Earth around the Sun	29600
Sun around galaxy	230000
Light (Electromagnetic Wave)	300000000

CRITICAL THINKING?

1. What is the displacement of the moving body, shown below?





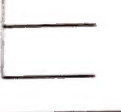

- A. 6 m
B. 7 m
C. 8 m
D. 23 m
2. A body moves from point P to point Q with a speed of 6ms^{-1} along a straight line then from Q to P with a speed of 4ms^{-1} . What is its average speed over the entire trip?
- A. 4ms^{-1}
B. 4.8ms^{-1}
C. 5ms^{-1}
D. 5.5ms^{-1}
3. In the given diagram, the coach moves from position A to B to C to D. What is the coach's resulting displacement and distance of travel?



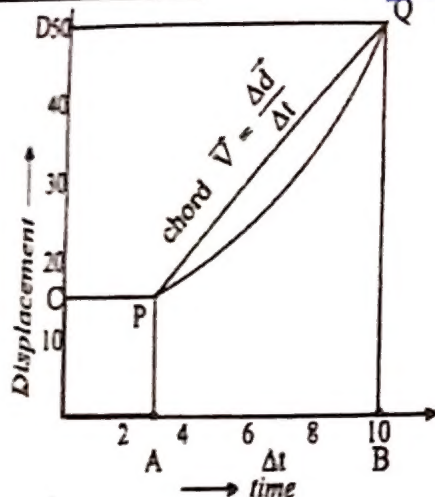
- A. 55yards left and 90 yards
B. 50yards left and 95 yards
C. 55yards left and 95 yards
D. 45yards left and 95 yards

DISPLACEMENT-TIME GRAPH

- The velocity at any instant is found by the slope of the displacement – time graph.
- For a body at rest, the displacement – time graph is a straight line parallel to the time axis.
- For a body moving with constant velocity, the displacement time graph is a straight line inclined to time axis.
- If slope increases, the body speeds up and if slope decreases, the body slows down.

Graph Shape	Slope	Velocity
	Constant positive	$a = 0$ Velocity is constant $\text{slope} = \frac{\Delta d}{\Delta t} = v$ $\text{slope} = \tan \theta = v$
	positive	Velocity is increasing $a = +ve$
	Constant (zero)	Velocity is zero
	+ve decreasing negative ?	$a = -ve$ Velocity decreases

For your Information



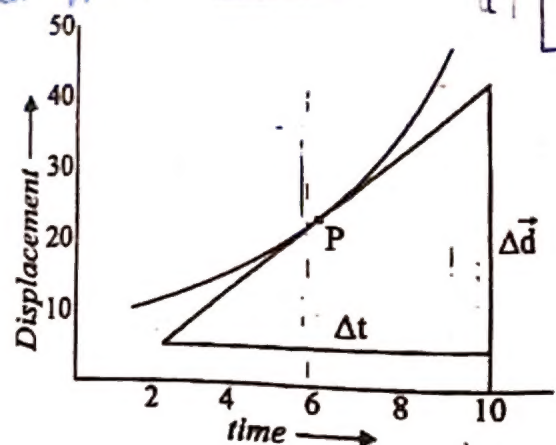
The chord in displacement-time graph shows average velocity of a moving body

ACCELERATION

Rate of change of velocity is known as acceleration. It is a vector quantity. If $\Delta \vec{v}$ is the change in velocity in time Δt , then acceleration is given as

$$a = \frac{\Delta \vec{v}}{\Delta t}$$

Acceleration may be positive or negative. Negative acceleration is termed as retardation. It indicates that the velocity of the body is decreasing with time.



A point P in Displacement-time graph shows instantaneous velocity of a moving body

Uniform Acceleration → $\Delta v / \Delta t$ remains constant.

If the velocity of a body changes by same amount in same interval of time, then the acceleration of the body is known as uniform acceleration.

- If a body travels with uniform acceleration a_1 for a time interval t_1 and with uniform acceleration a_2 for a time interval t_2 , then the average acceleration $\bar{a} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$

Instantaneous Acceleration

Acceleration of a body at a particular instant is known as instantaneous acceleration. It is obtained from average acceleration as Δt is made smaller and smaller till it approaches zero. Mathematically, instantaneous is the limit of the ratio of the change in velocity Δv to the time interval Δt as Δt following the instant t approaches zero.

$$\bar{a}_{\text{ins}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

The change in velocity can occur due to change in speed or in direction or in both.

Variable Acceleration:

The acceleration of a body is said to be variable if its velocity changes with time in terms of magnitude or direction or both. The variable acceleration is also called non-uniform acceleration.

GRAPHICAL REPRESENTATION OF ACCELERATION WITH VELOCITY TIME GRAPH

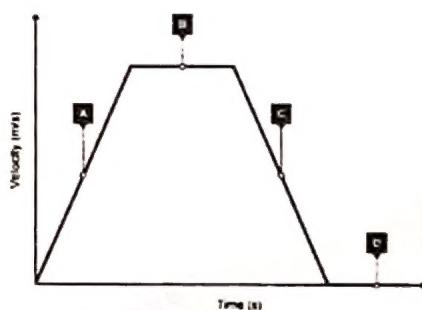
- The velocity at any instant is found by the slope of the displacement time graph. The slope of velocity time graph gives acceleration i.e. → change in magnitude

$$a = \frac{\Delta v}{\Delta t} = \tan \theta = \text{Slope of graph.}$$

→ change in magnitude + direction
centripetal acceleration

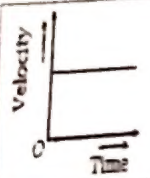
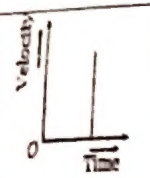
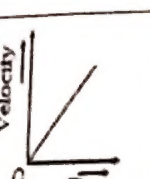

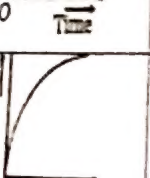
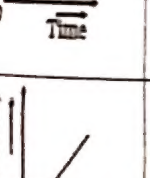
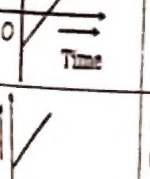
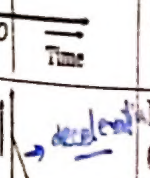
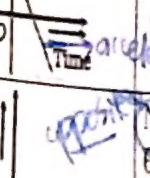
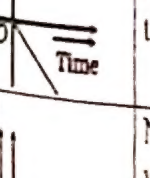
- The area under the v-t graph gives the distance covered by object.
- For a body moving with constant velocity, the velocity – time graph is a straight line parallel to time axis.
- For a body moving with constant acceleration, the velocity – time graph is straight line inclined to time axis.

For Your Information:

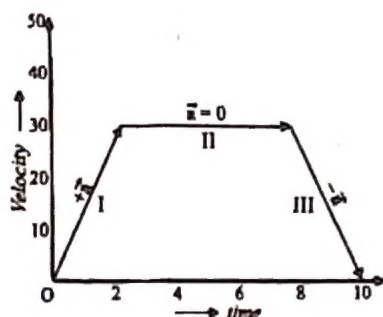
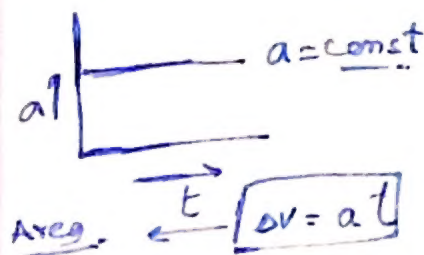


Section of graph	Gradient	Velocity	Acceleration
A	positive	increasing	positive
B	zero	constant	zero
C	negative	decreasing	negative
D	zero	zero (at rest)	zero

Various velocity - time graphs and their interpretation

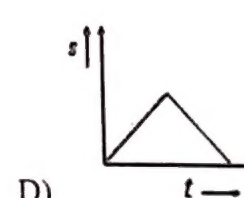
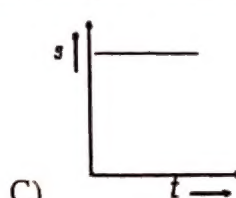
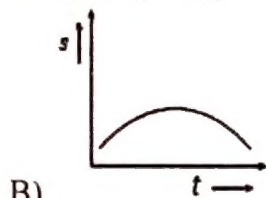
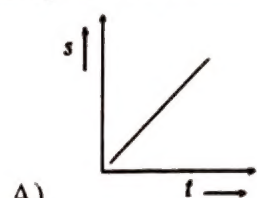
	$\theta = 0, a = 0, v = \text{constant}$ i.e., line parallel to time axis represents that the particle is moving with constant velocity. $\text{Area} = S = vt$ $\text{Slope} = 0, a = 0$
	$\theta = 90^\circ, a = \text{infinite}, v = \text{increasing}$ i.e., line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible
	$\theta = \text{constant}$, so $a = \text{constant}$ and v is increasing uniformly with time i.e., line with constant slope represents uniform acceleration of the particle. $S = v_0 t + \frac{1}{2} a t^2$
	θ increasing so acceleration increasing i.e., line bending towards velocity axis represent the increasing acceleration in the body. $\text{Slope} = \text{increasing}$ $a = +ve \text{ and increasing}$
	θ decreasing so acceleration decreasing i.e. line bending towards time axis represents the decreasing acceleration in the body. $\text{decreasing acceleration}$
	Positive constant acceleration because θ is constant and $< 90^\circ$ but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and $< 90^\circ$ but initial velocity of particle is positive
	Negative constant acceleration because θ is constant and $> 90^\circ$ but initial velocity of the particle is positive $\text{deceleration but in opposite direction}$
	Negative constant acceleration because θ is constant and $> 90^\circ$ but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and $> 90^\circ$ but initial velocity of the particle is negative.

For your Information



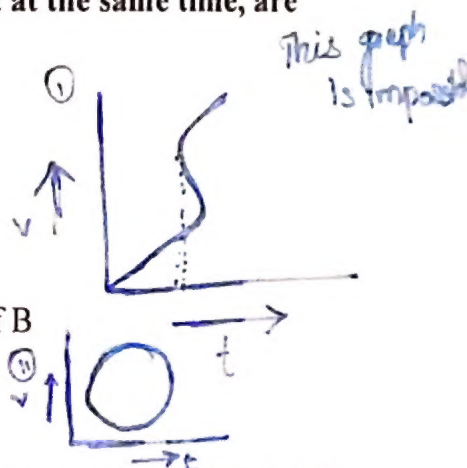
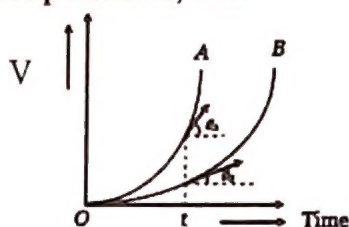
Velocity-time graph represents acceleration, uniform acceleration and deceleration.

Example: Which of the following graph represents uniform motion



Solution: A) When distance time graph is a straight line with constant slope than motion is uniform.

Example: Velocity-time graphs of two cars which start from rest at the same time, are shown in the figure. Graph shows, that

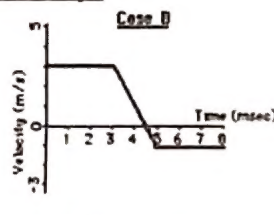
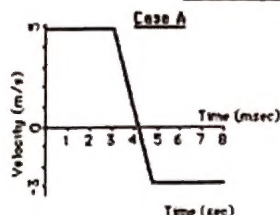


- A) Initial velocity of A is greater than the initial velocity of B
- B) Acceleration in A is increasing at lesser rate than in B
- C) Acceleration in A is greater than in B
- D) Acceleration in B is greater than in A

Solution: C) At a certain instant t slope of A is greater than B ($\theta_A > \theta_B$), so acceleration in A is greater than B

CRITICAL THINKING?

4. In the diagram given below in which case the acceleration is maximum
Velocity-Time Graph



- A. Case B
- C. Case A

- B. In both cases same
- D. None of these

NEWTON'S LAWS OF MOTION

Newton's Laws of Motion

First Law of Motion (Law of Inertia): It states that every body continues to be in state of rest or of uniform motion along a straight line unless it is compelled to change that state by an applied force.

- This law qualitatively defines the force.
- The inability of the body to change its state is called inertia. So, it is also known as the law of inertia of Galileo.
- Inertia resists change in the state of motion of the body.

Second Law of Motion (Force and Acceleration)

The effect of an applied force on a body is to cause it to accelerate in the direction of the force. The acceleration is in direct proportion to the force and is inversely proportional to the mass of the body.

OR

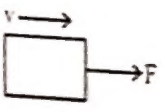
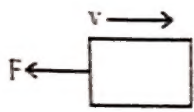
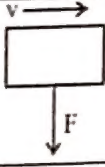
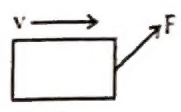
A force F acting on a body is equal to its change in momentum per second

$F \propto \text{Mass of the body} \times \text{Change in velocity per second} \propto ma$

$$\therefore F = k ma \quad (\text{where } k \text{ is a constant})$$

But $k = 1 \quad \therefore F = ma$

- **Weight:** The weight of a body is equal to the force with which the body is attracted by the earth towards its centre.

When force acts in direction of velocity	When force acts opposite to direction of velocity	When force acts perpendicular to direction of velocity	When force acts at some angle to the direction of velocity
			
Speed increases and direction of motion remain same	Speed decreases and direction of motion remain same	Only direction changes and magnitude of velocity remain same	Both magnitude and direction of velocity changes

CRITICAL CONCEPT

When a stone and leaf are dropped from a building simultaneously then why the stone reaches to the ground earlier?

Example: A man of mass 60 kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10 \text{ m/s}^2$).

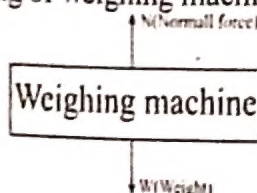
A) 600 N

B) 400 N

C) 200 N

D) 100 N

Solution: A) For calculating the reading of weighing machine, we draw diagram.



$$W = mg = 60 \times 10 = 600 \text{ N}$$

Example: A player caught a cricket ball of mass 150g moving at a rate of 20 m/s. If the catching process is completed in 0.1s, the force of the blow exerted by the ball on the hand of the player is equal to:

- A) 150 N B) 3 N C) 30 N D) 300 N

Solution: $F = \frac{mv}{t} = \frac{0.15 \times 20}{0.1}$

$F = 30 \text{ N}$

Third Law of Motion (Action and Reaction Forces)

It states that to every action, there is an equal and opposite reaction.

Action-reaction forces always occur in pairs.

If a body A exerts a force (action) on a body B, then B will exert an equal and opposite force (reaction) on A.

When a force acts on a body then the reaction acts normally to the surface of the body.

CRITICAL THINKING?

5. Newton's third law concerns the forces of interaction between two bodies. Which of the following statements relating to the third law is not correct?
- A. The two forces must be of the same type
 - B. The two forces must act on different bodies
 - C. The two forces are always opposite in direction
 - D. The two forces are equal and opposite so the bodies are in equilibrium

LINEAR MOMENTUM

"The idea of linear momentum was introduced by Newton who defined it as product of mass and velocity of an object".

$$\vec{p} = m \vec{v}$$

- Linear momentum is a vector pointing along velocity
- Linear momentum depends upon
 - $p \propto v$ $p \propto m$
- $p = 0$ if $v = 0$, how massive the body may be.

SI unit of linear momentum are kg m.s^{-1} or N s .

Dimension of momentum are $[MLT^{-1}]$

Newton's 2nd Law and Linear Momentum

Consider a body of mass m moving with an initial velocity \vec{v}_i . Suppose an external force

\vec{F} acts upon it for time t after which velocity becomes \vec{v}_f . The acceleration \vec{a} produced by this force is given by

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

By Newton's second law, the acceleration is given as

$$\vec{a} = \frac{\vec{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\vec{F}}{m} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

or

$$\vec{F} \times t = m \vec{v}_f - m \vec{v}_i = \text{Impulse} \quad \dots\dots\dots (i)$$

Where $m \vec{v}_i$ is the initial momentum and $m \vec{v}_f$ is the final momentum of the body.

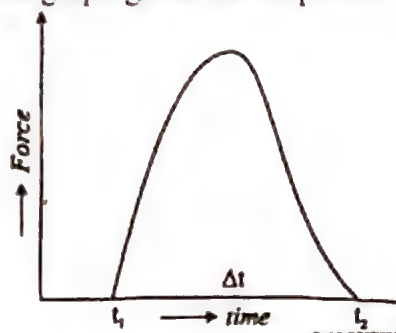
The equation (i) shows that change in momentum or Impulse is equal to the product of force and the time for which force is applied. This form of the second law is more general

than the form $\vec{F} = m \vec{a}$, because it can easily be extended to account for changes as a body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Equation (i)
$$\vec{F} = \frac{m \vec{v}_f - m \vec{v}_i}{t}$$

Thus, second law of motion can also be stated in terms of momentum as follows. Time rate of change of momentum of a body equals the applied force.

- The area under Force and time graph gives the Impulse.



For Your Information:

Impulsive force is a force which acts on a body for a very short time.

For example :

- A bat hitting the ball
- The collision between two snooker balls
- Air bag in the automobiles have saved countless lives in accidents. The air bag increases the time interval during which the passenger is brought to rest, thereby decreasing the force on the passenger.

CRITICAL THINKING?

6. What is the momentum of a runner of mass 65 kg who covers a displacement of 100m in 40 sec?

- | | |
|----------------------------|----------------------------|
| A. 162 kg ms ⁻¹ | B. 140 kg ms ⁻¹ |
| C. 150 kg ms ⁻¹ | D. 155 kg ms ⁻¹ |

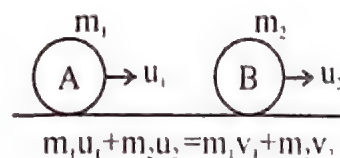
LAW OF CONSERVATION OF LINEAR MOMENTUM

Isolated system: In the absence of an external and unbalanced force, when two or more than two bodies are exerted the forces to one another during their collision is called isolated system.

In an isolated system $(p_i)_{\text{total}} = (p_f)_{\text{total}}$
 $m_1 v_1 + m_2 v_2 =$

The total linear momentum of an isolated system remains constant.

- Within the limits of experimental accuracy, it follows that the total momentum of A and B before collision = the total momentum after collision,
- If there is a system of particles free from external resultant force, $\vec{p}_1, \vec{p}_2, \vec{p}_3, \dots$ being the linear momentum of its individual particle then $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots = \text{constant}$
 Thus, for a system of particles the total linear momentum cannot change, unless an external resultant force acts on the system.



The principle of the conservation of linear momentum states that, if no external forces act on a system of colliding objects, the total momentum of the objects in a given direction before collision = total momentum in same direction after collision

Example: Two railway trucks of mass m and $3m$ move towards each in opposite directions with speeds $2v$ and v respectively. These trucks collide and stick together.

What is the speed of the trucks after the collision?

- A) $\frac{v}{4}$ B) $\frac{v}{2}$ C) v D) $\frac{5v}{4}$

Solution: A) Momentum before collision = momentum after collision

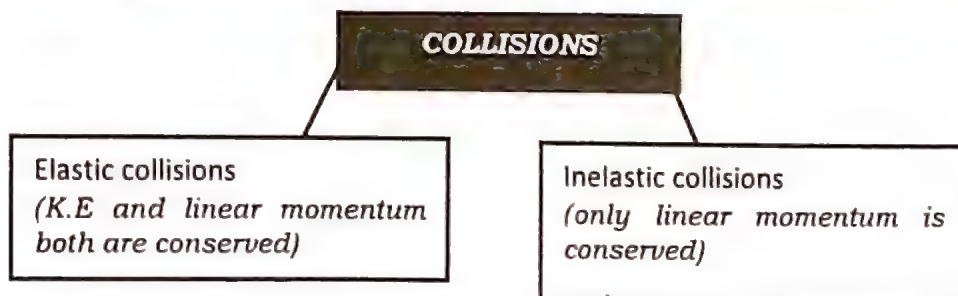
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$m(2v) + 3m(-v) = (m + 3m) v'$$

$$-mv = 4mv' \Rightarrow v' = -\frac{1}{4}v$$

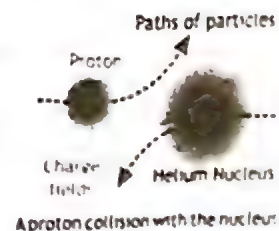
COLLISIONS

Collision is said to be taken place if some sort of interaction appears between bodies due to their closeness.



Elastic Collision in One Dimension

- In every collision the law of conservation of momentum is always obeyed.
- The collisions during which the kinetic energy is conserved, together with the momentum are called elastic collisions.
- If the collisions are not elastic, they are said to be inelastic.



- During inelastic collision, some part of the kinetic energy is converted into other form of energy, such as, heat energy, sound energy etc.
- If the colliding particles stick together and move as a single unit after collision, the collision is said to be perfectly inelastic collision.

Elastic Collision in one dimension:

- Elastic collision in one dimension is that one, in which colliding bodies do not deviate from their line of motion, after the collision.

- In case of two bodies undergoing elastic collision in one dimension, we have

$$v_1 + v'_1 = v_2 + v'_2 \quad \text{or} \quad v_1 - v_2 = -(v'_1 - v'_2).$$

Speed of approach = Speed of recession

After collision,

$$v'_1 = \frac{(m_1 - m_2)v_1}{(m_1 + m_2)} + \frac{2m_2v_2}{(m_1 + m_2)} \quad \text{or} \quad v'_2 = \frac{2m_1v_1}{(m_1 + m_2)} + \frac{(m_2 - m_1)v_2}{(m_1 + m_2)}$$

Elastic Collision In One Dimension Under Different Cases

Case I: If $m_1 = m_2$ & $v_2 \neq 0$

then $v'_1 = v_2$ & $v'_2 = v_1$

Case II: If $m_1 = m_2$ & $v_2 = 0$

then $v'_1 = 0$ & $v'_2 = v_1$

In both cases I & II due to the same masses of the colliding bodies their velocities after collision got interchanged.

Case III: If $m_1 \ll m_2$ & $v_2 = 0$

then $v'_1 = -v$ & $v'_2 = 0$

Case IV: If $m_1 \gg m_2$ & $v_2 = 0$

then $v'_1 = v_1$ & $v'_2 = 2v_1$

Collision Between a Body and the Floor or Wall

- In this case, the floor or wall is considered as a body of infinite mass with zero velocity.
- If the body strikes the surface normally, it is returned also normally either with its initial speed or with reduced speed. If the speed is not reduced the collision is elastic. The change in momentum in this case is $-2mv$

Examples:

- (1) A particle of mass having velocity ' v ' makes head on elastic collision with another particle of the same mass and initially at rest. The velocity of the first particle after the collision is

A) v C) $-v$

B) $\frac{v}{2}$ D) 0

Solution: If masses are same their velocities will alter, so velocity of 1st ball after collision = velocity of 2nd ball before collision = 0

- (2) A particle of mass m moving with velocity v strikes a stationary particle of mass $2m$ and sticks to it. The speed of the system will be?

A) $\frac{v}{2}$ B) $2v$ C) $\frac{v}{3}$ D) $3v$

Solution: $mv + 0 = (m + 2m)v'$

$$v' = \frac{mv}{3m} = \frac{v}{3}$$

CRITICAL THINKING?

7. In one dimensional elastic collision of two bodies of same masses, what will happen if the moving body collides with the mass which is initially at rest?
- Their velocities will be interchanged
 - Velocities of both bodies will be zero
 - Moving body will continue its motion
 - Moving body will come at rest and the mass at rest will start its motion

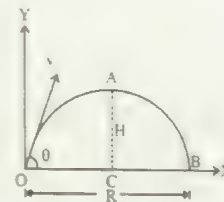
PROJECTILE MOTION**Projectile Motion**

- Projectile motion is a two-dimensional motion. The motion of the particle is constrained in a plane.
- When a particle is thrown obliquely near the earth's surface it moves in a parabolic path, provided the particle remains close to the surface of earth and the air resistance is negligible. This is an example of projectile motion.

Terms Used in Projectile Motion

Figure shows a particle projected from the point "O" with an initial velocity " v_i " at an angle θ with the horizontal.

- The point O is called the point of projection.
- The angle " θ " is called the "angle of projection".
- The distance OB is called the, horizontal range (R) or simply range, the vertical height AC is called maximum height (H) or Vertical Range.
- The total time taken by the particle in describing the path OAB is called the time of flight (T).
- Horizontal and vertical coordinates of projectile at time ' t ' are given as; $x = v_i \cos \theta t$ and $y = v_i \sin \theta t - \frac{1}{2}gt^2$
- Trajectory of projectile is parabola in the absence of air friction.
- If two identical balls are thrown simultaneously from same height, one vertically and other horizontally, then both falls to the earth simultaneously.

**Time of flight:**

- Time to reach maximum height is given as; $t = \frac{v_i \sin \theta}{g}$
- Total time of flight is given as; $T = \frac{2v_i \sin \theta}{g}$

Maximum Height:

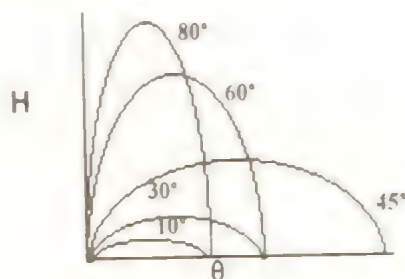
- Vertical range (height) is given as; $H = \frac{v_i^2 \sin^2 \theta}{2g}$

Horizontal range:

- Range (horizontal) is a distance between point of projection and point at which it comes back to its level of projection. It is given as;

$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{v_i^2 (2 \sin \theta \cos \theta)}{g}$$

- Maximum horizontal range is at angle $\theta = 45^\circ$ and given as: $R_{\max} = \frac{v_i^2}{g}$
- The relation between range and the height of the projectile is $R \tan \theta = 4H$
- With same initial velocity the range of projectile for two angles of projection will be equal if sum of the angles is equal to 90° i.e. $\theta_1 + \theta_2 = 90^\circ$
- Variation in the range and height with angle of projection is shown with the following sketch for same speed of projectile.



- When angle of projection is 76° then range and maximum height of projectile are equal to each other.

Application to Ballistic Missile

- An **un-powered** and **un-guided** missile is called ballistic missile
 - Friction of air effects the horizontal and vertical motion of the missile
 - Ballistic missiles are useful only for short ranges.
- Powered and remote control guided missiles are used for long ranges and precision.

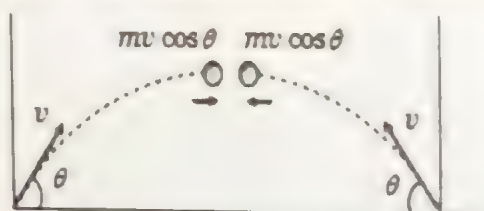
CRITICAL CONCEPT

How rockets accelerate in space? As there is no air in space to push against such that as a reaction rocket pushed forward.

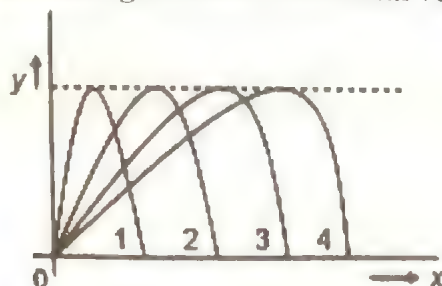
Example: Two equal masses (m) are projected at the same angle (θ) from two points separated by their range with equal velocities (v). The momentum at the point of their collision is

- A) Zero B) $2mv \cos \theta$ C) $-2mv \cos \theta$ D) None of these

Solution: A) Both masses will collide at the highest point of their trajectory with equal and opposite momentum. So net momentum of the system will be zero



Example: Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first



A) 1, 2, 3, 4

B) 2, 3, 4, 1

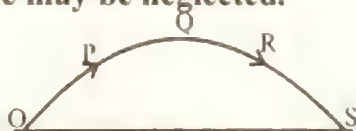
C) 3, 4, 1, 2

D) 4, 3, 2, 1

Solution: D) Range directly proportional to horizontal component of velocity. Graph 4 shows maximum range, so football possess maximum horizontal velocity in this case.

CRITICAL THINKING?

8. A body is projected with kinetic energy K at an angle of 60° with the horizontal. Its kinetic energy at the highest point of its trajectory will be
 - A. $2K$
 - B. K
 - C. $\frac{K}{2}$
 - D. $\frac{K}{4}$
9. Four projectiles are projected with the same speed at angles 20° , 35° , 60° and 75° with the horizontal. The range will be the longest for the projectile whose angle is
 - A. 20°
 - B. 35°
 - C. 60°
 - D. 75°
10. A projectile is launched at point O and follows the path OPQRS, as shown. Air resistance may be neglected.



Which statement is true for the projectile when it is at the highest point Q of its path?

- A. The horizontal component of the projectile's acceleration is zero
- B. The horizontal component of the projectile's velocity is zero
- C. The kinetic energy of the projectile is zero
- D. The momentum of the projectile is zero

TOPIC-2 **WORK AND ENERGY**

COURSE CONTENT

- Work
- Energy
- Kinetic energy
- Potential energy
- Gravitational potential energy
- Power
- Work Energy Principle
- Explain the work done against friction is dissipated as heat in the environment
- Implications of energy losses in practical devices

WORK

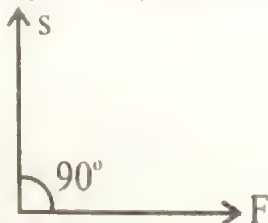
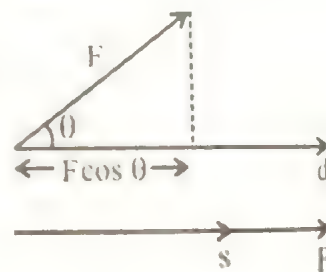
The work done by a force in displacing an object is defined as the product of the displacement and the component of the force in the direction of the displacement is called work.

$W = \text{Component of force in the direction of displacement} \times \text{displacement}$

$$W = (F \cos \theta) (d)$$

$W = F s \cos \theta$. F is force, s or d is displacement and θ is angle between F and s or d .

- If $\theta = 0^\circ$ Then $W = F s \cos 0^\circ$ or $W = F s$. It is the maximum value of work.
- If $\theta = 90^\circ$ Then $W = F s \cos 90^\circ$ or, $W = 0$, so work is zero



- If $\theta = 180^\circ$ Then $W = F s \cos 180^\circ$ or, $W = -F s$



So, work is negative and is said to be done or, the force.

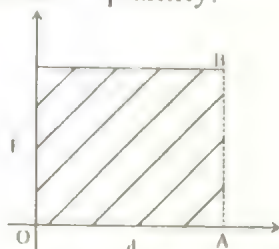
In S.I the unit of work is joule (J) and in C.G.S the unit of work is erg.

CRITICAL CONCEPT

The work done is 100% when the applied force is acting at angle 0 in the direction of displacement. what would be angle of applied force with displacement when work done is 50%

Work Done by Constant Force:

Graphically work can be obtained from force displacement graph. The area under this graph is work done. Work is a **scalar** quantity.



Work done = Area of rectangle = (OA) (AB) = (F) (d)

Work Done by a Variable Force

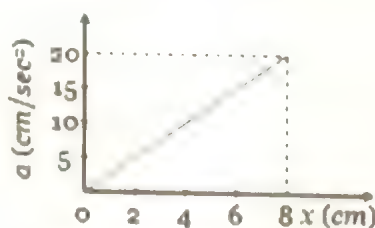
$$\text{Total Area} = \lim_{\Delta d \rightarrow 0} \sum_{i=1}^n F_i \cos \theta_i \Delta d_i = \text{work done}$$

Example: A box of mass 1 kg is pulled on a horizontal plane of length 1 m by a force of 8 N then it is raised vertically to a height of 2m, the net work done is

- A) 28 J B) 8 J C) 18 J D) None of above

Solution: A) Work done to displace it horizontally = $F \times s = 8 \times 1 = 8 \text{ J}$ Work done to raise it vertically $F \times s = mgh = 1 \times 10 \times 2 = 20 \text{ J}$ hence, Net work done = $8 + 20 = 28 \text{ J}$

Example: A 10 kg mass moves along x-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from $x = 0$ to $x = 8\text{cm}$?



- A) 8 J B) $8 \times 10^{-2} \text{ J}$ C) $16 \times 10^{-2} \text{ J}$ D) $4 \times 10^{-3} \text{ J}$

Solution: Work done on the mass = mass \times covered area between the graph and displacement axis on a-x graph

$$= 10 \times \frac{1}{2} (8 \times 10^{-2}) \times 20 \times 10^{-2} = 8 \times 10^{-2} \text{ J}$$

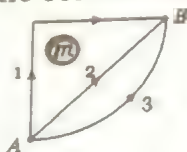
Conservative Field

- If a field satisfies following two conditions, it is said to be conservative.
- (i) Work done along a closed path is zero i.e. $W_{\text{total}} = 0$
- (ii) Work done is independent of path followed by body but depends on final and initial position of a body.

Example: Electric field, Gravitational field.

- Frictional force is non-conservative force. Other non-conservative forces are propulsion force on rocket, force of a motor, tension in string etc.
- Spring force $F = kx$ is conservative force.
- Conservative field and conservative force has the property of storing energy in the system. This energy is known as P.E of the system.

Example: If W_1 , W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m , find the correct relation



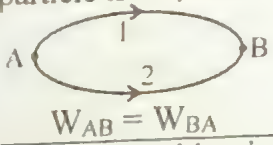
A) $W_1 = W_2 = W_3$

B) $W_1 > W_2 > W_3$

C) $W_1 < W_2 < W_3$

D) none of these

Solution: A) As gravitational field is conservative in nature. So work done in moving a particle from A to B does not depend upon the path followed by the body. It always remains same.

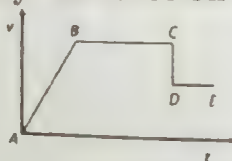
Conservative forces	Non-conservative forces
1. The work done by these forces in carrying a particle around a closed path is zero i.e., $W_{\text{total}} = 0$.	The work done by these forces in carrying a particle around a closed path is not zero i.e., $W_{\text{total}} \neq 0$
2. The work done by these forces in displacing a particle does not depend on the path along which the particle is displaced.  $W_{AB} = W_{BA}$	The work done by these forces depends upon the path along which the particle is displaced. In this case $(W_{AB}) \neq (W_{BA})$
3. Under these forces the kinetic energy of the particle remains constant $K.E_i = K.E_f$ e.g., central forces, gravitational force elastic force, Lorentz force, electrostatic force, magnetic force etc.	Under these forces the kinetic energy of the particle changes $K.E_i \neq K.E_f$ e.g., frictional force, retarding force, viscous force, magnetic force due to an electric current etc. (all these are velocity dependent forces)

For Your Information:

We can calculate the work done by a force on an object, but that force is not necessarily the cause of the displacement. For example, if you lift a body, work is done on the object by the gravitational force, although gravity is not the cause of the object moving upward

CRITICAL THINKING?

1. A man pulling a bag with force of 15N at angle 60° with horizontal plane. If bag covers a distance of 10m, then work done by the man is
 A. 50J
 B. 75J
 C. 150J
 D. 100J
2. The adjoining diagram shows the velocity versus time plot for a particle. The work done by the force on the particle is positive from



A. B to C

B. D to E

C. A to B

D. C to D

ENERGY

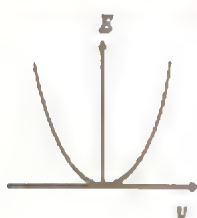
Energy of a body is defined as its ability to do work. Units of energy are the same as that of work. The SI unit of energy is joule. Other units of energy are erg, foot-pound and kilowatt-hour etc. energy occurs in many forms such as mechanical, electrical, chemical, nuclear, magnetic, heat and elastic energy etc. energy possessed by a body is basically of two types: Kinetic energy and Potential energy.

KINETIC ENERGY

$$\text{K.E} = \frac{1}{2}mv^2$$

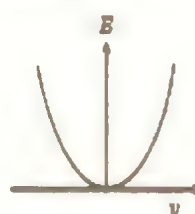
- Relation between linear momentum (p) and kinetic energy: $p^2 = 2m \text{ K.E.}$
- For two bodies having equal momentum $\frac{\text{K.E.}_1}{\text{K.E.}_2} = \frac{m_2}{m_1}$
- For two bodies having equal kinetic energies: $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

Graphs of kinetic Energy



$$E \propto v^2$$

$$m = \text{constant}$$



$$E \propto p^2$$

$$m = \text{constant}$$



$$B \propto \frac{1}{M}$$

$p = \text{constant}$

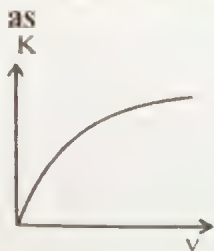


$$p \propto \sqrt{E}$$

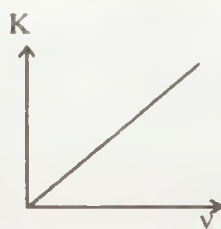
$$m = \text{constant}$$

CRITICAL THINKING?

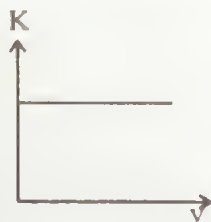
3. A bullet of mass 20g is fired with velocity of 2000 ms^{-1} , the K.E of the bullet is
A. 2000 J B. 4000 J
C. 20000 J D. 40000 J
4. The graph of kinetic energy (K.E) of the body versus velocity (v) is represented by



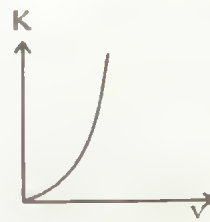
A.



B.



C.



D.

GRAVITATIONAL POTENTIAL ENERGY

- The potential energy of a body is defined as the energy possesses by the body by virtue of its position or configuration.
- Gravitational Potential Energy of a body of mass m at height h from surface of earth:
 $P.E = mgh$

CRITICAL THINKING?

5. A body of mass 100 g is raised vertically from surface of earth in a gravitational field. The P.E of the body at height 100 m is
- A. 0.98 J B. 9.8 J
C. 98 J D. 980 J

POWER

Power (P)

$$\text{Power (P)} = \frac{\text{Work}}{\text{Time}} \quad \text{or} \quad P = \frac{W}{t}$$

- If the point of application of a force F moves with a velocity v , then Power $P = \vec{F} \cdot \vec{v}$ or, $P = Fv \cos \theta$, θ is angle between \vec{F} and \vec{v} .
- Instantaneous power is given as; $P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}$. If $\langle P \rangle = P_{\text{avg}}$, then body does work at constant rate.
- Unit of power is watt defined as; 1 W = 1 J/1 sec
- Dimensionally power is equivalent to $[ML^2T^{-3}]$
- 1 h.p = 746 watt = 550 foot pound/sec
- Commercial unit of electricity is kW h since, 1 kW h = 3.6×10^6 J
- Kilowatt hour (kW h) is also known as B.O.T.U (Board of trade unit)
B.O.T.U. = 1 kW h = 3.6×10^6 joule

Example: From a water fall, water is falling at the rate of 100 kg/s on the blades of turbine.

If the height of the fall is 100m then the power delivered to the turbine is approximately equal to

- A) 100 kW C) 1 kW
B) 10 kW D) 1000 kW

Solution: A)

$$\text{Power} = \frac{\text{work done}}{t} = \frac{mgh}{t} = 100 \times 10 \times 100 = 10^5 \text{ watt} = 100 \text{ kW} \left[\text{As } \frac{m}{t} = 100 \frac{\text{kg}}{\text{sec}} (\text{given}) \right]$$

CRITICAL THINKING?

6. What is the power of an electric motor when it consumes energy of 9×10^3 J in 3 s?
A. 1 hp B. 2 hp C. 3 hp D. 4 hp
7. A man does a given amount of work in 10 sec. Another man does the same amount of work in 20 sec. The ratio of the output power of first man to the second man is
A. 2/1 B. 1/2
C. 1 D. 4/1

WORK ENERGY PRINCIPLE

$$\text{Work done} = \text{Change in kinetic energy} = K.E_f - K.E_i \quad \text{or} \quad W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- If a car stopped by applying brakes, then the stopping distance $s = \frac{mv^2}{2F} \Rightarrow s \propto v^2$

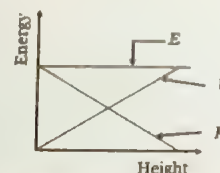
Here F is the force applied by brakes and v is the velocity of car.

Elastic P.E.

$$E.P.E = \frac{1}{2} k x^2$$

Energy height graph:

When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its potential energy (U) is zero. As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but throughout the complete motion total energy remains constant as shown in the figure.



CRITICAL CONCEPT

If a spring is compressed, then work done on it equals the increase in its elastic potential energy.

WORK DONE AGAINST FRICTION IS DISSIPATED AS HEAT IN THE ENVIRONMENT

Conservation of Energy

Energy cannot be destroyed. It can be transformed from one form into another, but total amount of energy remains constant.

When a cup is dropped the P.E changes to K.E, but on striking the ground, the K.E changes to heat and sound but total energy at each instance is always conserved.

Kinetic energy + Potential energy = constant

Work done against friction dissipated as heat in the environment

- If a body is dropped from height 'h' to earth's surface in **absence of air**, then;
Loss in P.E. = Gain in K.E.

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$

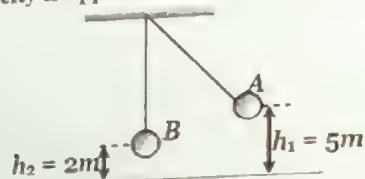
- If a body is dropped from height 'h' to earth's surface in **presence of air**, then;
Loss in P.E = gain in K.E + work done against air.

$$mgh = \frac{1}{2}mv^2 + fh$$

- If a body is thrown vertically upward in gravitational field in the **presence of air**, then;
Loss in K.E = gain in P.E + work done against air.

$$\frac{1}{2}mv^2 = mgh + fh$$

Example: A boy is sitting on a swing at a maximum height of 5m above the ground. When the swing passes through the mean position which is 2m above the ground its velocity is approximately



- A) 7.6 m/s
B) 9.8 m/s

- C) 6.26 m/s
D) None of these

Solution:

$$V = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 9.8 \times 3} = 7.6 \text{ ms}^{-1}$$

CRITICAL THINKING?

8. A trolley runs from P to Q along a track. At Q, its potential energy is 50 kJ less than at P.



At P, the kinetic energy of the trolley is 5 kJ. Between P and Q the work that the trolley does against friction is 10 kJ. What is the kinetic energy of the trolley at Q?

- A. 35 kJ
B. 55 kJ
C. 45 kJ
D. 65 kJ

IMPLICATIONS OF ENERGY LOSSES IN PRACTICAL DEVICES

- Mechanical efficiency is the ratio of work output to work input.
- The efficiency of an ideal machine is 100 percent but an actual machine's efficiency will always be less than 100%.
- If a machine moves a load W through a distance h then the useful work done by the machine is called output.

$$\text{Output} = \text{Load} \times \text{distance } h \text{ through which the load moves} = F_{out} \times D_{out}$$

- If an effort F_{in} acts through a distance D_{in} then the work done on the machine is called input.

$$\text{In put} = \text{Effort force} \times \text{Effort distance.}$$

$$\text{In put} = F_{in} \times D_{in}$$

- The ratio of output to the input of a machine is called its efficiency.

$$\begin{aligned} \text{Mathematically} \quad \text{Efficiency} &= \frac{\text{out put work}}{\text{in put work}} \\ &= \frac{\text{Load force} \times \text{Load distance}}{\text{Effort force} \times \text{Effort distance}} \end{aligned}$$

$$\text{Efficiency} = \frac{F_{out} \times D_{out}}{F_{in} \times D_{in}}$$

The equation for percentage efficiency is

$$\begin{aligned} \text{Percentage Efficiency} &= \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{W_{out}}{W_{in}} \times 100\% \\ &= \frac{F_{out} \times D_{out}}{F_{in} \times D_{in}} \times 100\% \dots 4.7 \end{aligned}$$

- Even a very efficient device will waste some of its input energy in the form of heat due to the friction forces between different parts of machine.
- An incline is used as a simple machine. Which is a flat surface tilted at an angle. Which is commonly used to load truck, planes and trains.

Table	
Practical devices	Efficiency
Petrol heat engine	(25-30)%
Diesel engine	(34-40)%
Steam locomotive	(35-40)%
Incandescent lamp	5%
Fluorescent lamp	20%
Steam turbine	(34-46)%
Air craft gas turbine	36%
Nuclear power plant	(30-35)%
Fossil fuel power plant	(30-40)%
Electric generator	(70-98)%
Electric motor	(50-92)%
Dry cell battery	90%
Battery	90%
Home coal furnace	55%

FOR YOUR INFORMATION

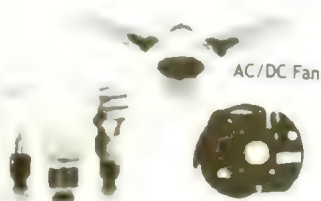
Efficiency of some electrical equipment's: LED light bulbs have been introduced to replace ordinary light bulbs, as they are much more efficient. Let's take a look at a standard 50-watt. The energy consumption to use a light bulb like this would cost about 1278 Rs in a year. An LED, running over the course of 1 year would cost only 260Rs to operate.

Using these causes less energy to be wasted as heat. Recently developed, AC/DC fans can operate on less energy while producing a high airflow. In fact, they can cut down your power consumption by up to 65% and can operate on solar panel. AC/DC fans are designed to run on 12V and consume around 26-35W. Ordinary Fan consume 75watt while AC/DC fans consume about 35 to 40watt so AC/DC fans are more efficient.

Example

A machine needed 1000J of energy to raise a 10kg block at a distance of 6.0m. What is the machine efficiency?

- (A) 40% (B) 49%
(C) 59% (D) 50%



AC/DC Fan

Solution: First, find the work done to raise the block. $W = mgh$

$$= 10\text{kg} \times 9.8\text{m/s}^2 \times 6.0\text{m} = 588\text{J}$$

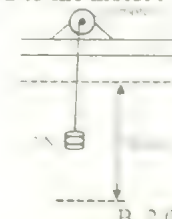
$$\text{Efficiency } \eta = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{588}{1000} \times 100\% = 58.8\%$$

Pulleys are machines used to lift heavy loads. Modern cranes are complex pulley system

$\eta = 59\%$ Answer

CRITICAL THINKING?

9. A small electric motor is used to raise a weight of 2.0 N through a vertical height of 80 cm in 4.0 s. The efficiency of the motor is 20%. What is the electrical power supplied to the motor?



- A. 0.080 W
C. 0.80 W

- B. 2.0 W
D. 200 W

TOPIC 3

ROTATIONAL & CIRCULAR MOTION

COURSE CONTENT

- Angular displacement (Revolution, Degree, Radian)
- Angular velocity
- Angular acceleration
- Angular momentum
- Angular impulse
- Angular momentum and angular velocity
- Angular momentum and angular acceleration
- Angular momentum and angular velocity

ANGULAR DISPLACEMENT (REVOLUTION, DEGREE, RADIAN)

Circular Motion

- Angular displacement in circular path is called circular motion
- During circular motion, the direction of position vector changes continuously. Its magnitude remains constant which is equal to r (radius of circular path)
- In circular motion, the direction of velocity change at every point but its magnitude remains constant
- For one complete revolution, the angular displacement is 2π and time taken is T time period. So angular velocity $\omega = \frac{2\pi}{T}$

- Speed, kinetic energy and angular momentum remain constant in circular motion

Angular Motion

Circular motion is a type of motion which is called angular motion.

Angular Displacement

It is the angle swept by the radius line during circular motion of a particle moving from some initial point to some final point.

- Angular displacement has direction along axis of rotation and can be determined by right hand rule.

Radian

- SI unit of angular displacement is radian
- One radian is an angle made by an arc at the center, whose length is equal to the radius of circle
- Definition of radian gives following useful relations $S = r\theta$
 $\theta = \frac{S}{r}$ rad = 0.0174 rad, $1 \text{ rad} = 57.3^\circ$
- Angular displacement is equal to radian in degree covered by body during circular motion
- Now SI units are also used which are "degree" and "rev"

Topic-3

Rotational & Circular Motion

For Your Information

- Angle swept by a minute hand in one complete rotation is 2π
- Angle swept by minute hand in one minute is π
- Angle swept by minute hand in 5 minutes is $5 \times \pi = 5\pi$

Q.1. Ali goes around a circular track that has a diameter of 20m. If he runs around the entire track for a distance of 160m, what is his angular displacement?

- Ali goes around a circular track that has a diameter of 20m. If he runs around the entire track for a distance of 160m, what is his angular displacement?
 A. 16 rad
 B. 8 rad
 C. 5 rad
 D. 3 rad
- 85.95 degree in terms of radian is
 A. 1.5 rad
 B. 1 rad
 C. 1.5 rad
 D. 2 rad

ANGULAR VELOCITY

Rate of change of angular displacement is called angular velocity.

$\omega = \frac{d\theta}{dt}$, usually not a vector quantity or $\omega = \frac{1}{2\pi} \frac{d\theta}{dt}$ where θ is angular displacement.

- Tangential and angular velocities are related as $v = \omega r$
- SI unit of angular velocity is rad s^{-1}
- The magnitude of an angular velocity is called the angular speed which is represented by ω

Example: The angular velocity of seconds hand of a watch will be

- $\frac{\pi}{30} \text{ rad/sec}$
- $30\pi \text{ rad/sec}$
- $60\pi \text{ rad/sec}$
- $\frac{\pi}{60} \text{ rad/sec}$

Solution: We know that seconds hand completes one revolution in 60 seconds.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/sec}$$

Angular Acceleration

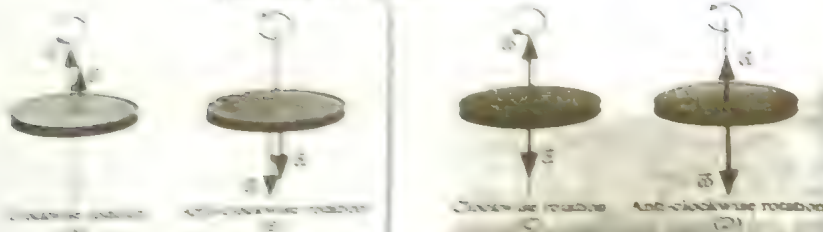
- The rate of change of angular velocity is called angular acceleration. If ω_1 is angular velocity at time t_1 and angular velocity ω_2 at time t_2 then

$$\text{Angular acceleration} = \alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

Analogy Between Translatory Motion And Rotatory Motion

Linear velocity, $v = \frac{\Delta d}{\Delta t}$	Angular velocity, $\omega = \frac{\Delta \theta}{\Delta t}$
Linear displacement, d	Angular displacement, θ
Acceleration or linear acceleration, $a = \frac{\Delta v}{\Delta t}$	Angular acceleration, $\alpha = \frac{\Delta \omega}{\Delta t}$
Mass, m	Moment of inertia, $I = mr^2$
Linear momentum, $p = mv$	Angular momentum, $L = I\omega$
Impulse, I or $J = \vec{F} \times \Delta t$	Angular impulse = $\vec{\tau} \times \Delta t$
Force, $F = ma = \frac{\Delta p}{\Delta t}$	Torque, $\vec{\tau} = I\vec{\alpha} = \frac{\Delta L}{\Delta t}$
Work, $W = Fd$	Rotational work, $W = \tau\theta$
Kinetic energy, $KE = \frac{1}{2}mv^2$	Kinetic energy of rotation, $KE = \frac{1}{2}I\omega^2$
Newton's laws in linear motion	Newton's laws in rotational motion
First law As $F = 0$ then $v = \text{constant}$ or $v = 0$	First law As $\tau = 0$ then $\omega = \text{constant}$ or $\omega = 0$
Second Law $\vec{F} = m\vec{a}$	Second Law $\vec{\tau} = I\vec{\alpha}$
Third Law $\vec{F}_1 = -\vec{F}_2$	Third Law $\vec{\tau}_1 = -\vec{\tau}_2$

Direction of Angular Acceleration



When the angular velocity is increasing, the angular acceleration vector points in the same direction as the angular velocity, as shown in Fig. (A) and (C).

When the angular velocity is decreasing, the angular acceleration vector points in the opposite direction to the angular velocity, as shown in Fig. (B) and (D).

CRITICAL THINKING?

3. A body moves with constant angular velocity in a circle. Magnitude of angular acceleration is
- (A) ω (B) Constant
(C) 0 (D) ω^2

RELATION BETWEEN LINEAR AND ANGULAR VARIABLES

- Relationship between linear and angular displacement is $s = r\theta$
where s is linear displacement
- Relationship between linear and angular velocity is $v = r\omega$
where v is linear velocity
- Relationship between linear and angular acceleration is $a = r\alpha$
where a is linear acceleration

CENTRIPETAL FORCE (CENTRIPETAL ACCELERATION)

The force required to bend a straight-line path of a body into the circular path is called centripetal force.

- If the centripetal force is removed from the rotating object, it will follow a straight-line motion confined on the tangent to that circular path.
- In vector form, centripetal force and acceleration can be written as:

$$\vec{F}_c = -m\omega^2 \vec{r} = -m\omega^2 r \hat{r} = -\left(\frac{mv^2}{r}\right) \hat{r} = -\left(\frac{mv^2}{r}\right) \frac{\vec{r}}{r} \quad \text{or} \quad |\vec{F}_c| = \frac{mv^2}{r} = m\omega^2 r$$

$$\vec{a}_c = -\omega^2 \vec{r} = -\omega^2 r \hat{r} = -\left(\frac{v^2}{r}\right) \hat{r} = -\left(\frac{v^2}{r}\right) \frac{\vec{r}}{r} \quad \text{or} \quad |\vec{a}_c| = \frac{v^2}{r} = \omega^2 r$$

- Work done by centripetal force is zero.
- Centripetal and centrifugal force are not action & reaction pair as they do not act on each other because they don't act on same body.

Example: An object of mass of 2 kg rotates at constant speed in a horizontal circle of radius 5 m. The time for one complete revolution is 1 s. What is the magnitude of the resultant force acting on the object?

- (A) $10\pi^2$ N
(B) π^2 N
(C) $40\pi^2$ N
(D) $\frac{40\pi^2}{9}$ N

Solution: (B) $F = \frac{mv^2}{r}$

$$v = \frac{2\pi r}{T}$$

$$F = \frac{2\pi^2 m}{T^2} = \frac{40\pi^2}{9} \text{ N}$$

CRITICAL THINKING?

4. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is
A. 250 N
B. 750 N
C. 1200 N
D. 1000 N



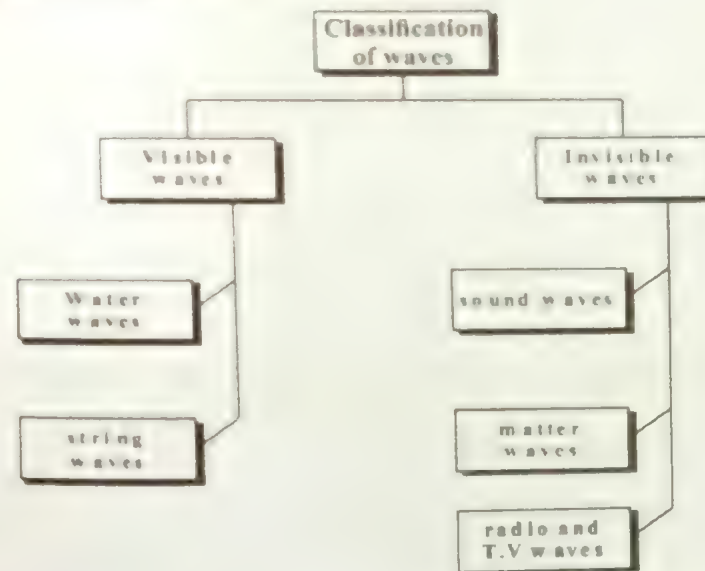
COURSE CONTENT

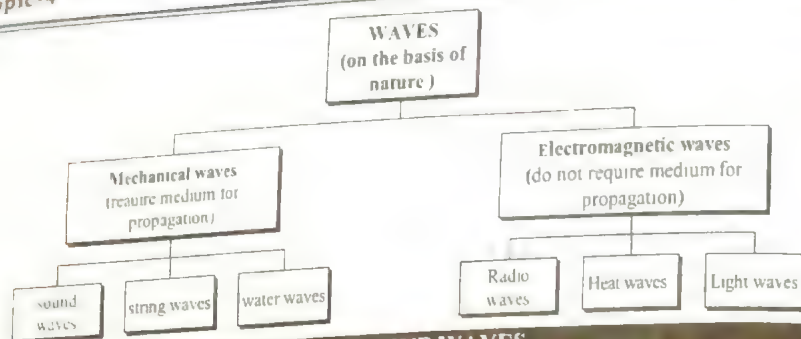
- Progressive wave (Crest, Trough, amplitude, wavelength, time period and frequency)
- Types of progressive waves (Transverse waves, Longitudinal waves)
- Speed of sound in air
- Principle of superposition/ superposition of sound waves
- Stationary waves/ standing waves
- Stationary waves in a stretched string (fundamental frequency and harmonics)
- Doppler effect
- Simple harmonic motion (SHM)
- Characteristics of simple harmonic motion
- Instantaneous displacement
- Amplitude
- Vibration
- Time period
- Frequency

Introduction

- Waves are disturbances that travel through a medium
- Waves transport energy without transporting matter

Classification of Waves



**PROGRESSIVE WAVES**

- Progressive wave or Traveling wave is that which propagates or distributes its pulses in space along specific direction. e.g.
 - Waves in a string
 - Waves on a water surface

Types of Progressive Waves

- On the basis of vibration of the particle, waves are classified in two types
 - Transverse Waves
 - Longitudinal Waves

(a) Transverse Waves

- The particles of the medium vibrate at right angles to the direction of propagation of the wave.
- Crests and troughs are produced.
- It is because a liquid surface has property of surface tension which resists any deformation of shape.
- Transverse wave is not produced or possible in gases.

Crest:

A crest point on a wave is the maximum value of upward displacement within a cycle. A crest is a point on a surface wave where the displacement of the medium is at a maximum.

Trough:

A trough is the opposite of a crest, so the minimum or lowest point in a cycle.

(b) Longitudinal Waves

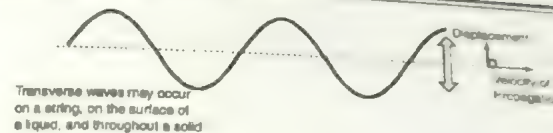
- The particles of the medium vibrate along the direction of propagation of the wave.
- Compressions and rarefactions are produced.
- Longitudinal waves are possible in all media i.e., solid, liquid and gas.

PERIODIC WAVES

- Periodic waves are those, which are repeated in regular interval of time.
- Periodic wave may be transverse or longitudinal.

Transverse Periodic waves

- For transverse waves the displacement of the medium is perpendicular to the direction of propagation of the wave. A ripple in a pond and a wave in a string are easily visualized as transverse waves



- Transverse waves cannot propagate in a gas or a liquid because there is no mechanism for driving motion perpendicular to the propagation of the wave.
- In fluids, transverse waves die out very quickly and usually cannot be produced at all.
- In a transverse periodic wave time interval equal to time period, a particle in the wave travels a distance equal to **wavelength**.
- For all waves $v = f\lambda$.

Do you know

The waves transport both energy and momentum in a medium.

Characteristics of Wave Motion**(i) Frequency (f)**

The number of waves which pass a point per unit time is called the frequency of the wave motion.

(ii) Wavelength (λ)

It is shortest distance between two consecutive points in the same phase.

(iii) Time Period (T)

Time taken to complete one vibration is called time period.

(iv) Amplitude (A)

The maximum displacement of a vibrating particle from mean position is called its amplitude

(v) Wave Velocity (v)

The distance traveled by the wave in one second is defined as its velocity.

- The velocity of the particles of the medium is different from the velocity of the wave

- Wave velocity = frequency \times wavelength $v = f\lambda$

- The relation $v = f\lambda$ holds good for any type of wave motion - transverse or longitudinal
- When a given wave passes from one medium to the other, its frequency does not change.

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

- The particles in the wave separated by a distance which is integral multiple of λ i.e. $n\lambda$ are in phase with each other.

- The particles separated by a distance which is odd multiple of $\frac{\lambda}{2}$ i.e.

$$\left(n + \frac{1}{2}\right)\lambda = (2n+1)\frac{\lambda}{2} \text{ are out phase to each other.}$$

Example: A sound wave of frequency 400 Hz is travelling in a gas at a speed of 320 m s^{-1} . What is the phase difference between two points 0.2 m apart in the direction of travel?

A) $\frac{\pi}{4} \text{ rad}$ B) $\frac{\pi}{2} \text{ rad}$ C) $\frac{2\pi}{5} \text{ rad}$ D) $\frac{4\pi}{5} \text{ rad}$

Solution: B) As $v = f\lambda$ so $\lambda = \frac{v}{f} = \frac{320}{400} = 0.8 \text{ m}$, Phase difference = $\frac{2\pi x}{\lambda} = \frac{0.2}{0.8} \times 2\pi = \frac{\pi}{2} \text{ rad}$

Q. A wave is traveling toward the right as shown. Which letter correctly labels the amplitude of the wave?

LONGITUDINAL PERIODIC WAVES

The direction of vibration of a medium is parallel to the propagation of the wave. Sound waves in air are longitudinal waves.



SPEED OF SOUND IN AIR

- Longitudinal waves in air are sound waves ($\lambda = 1\text{m}$)
- They travel in all directions in the direction of wave
- A vibrating body is a production of sound
- Medium for propagation of sound
- Direction of vibration of sound
- Sound waves are longitudinal waves having three dimensional propagations in air.
- Longitudinal sound waves consist of compressions and rarefactions.
- Compression is a region where crowding of particles of medium is maximum.
- Rarefaction is region where crowding of particles of medium is minimum.
- Sound waves produce Reflection, Refraction, Diffraction, Interference but not polarization because sound waves are longitudinal.

Gases (20°C)		Liquids (25°C)		Solids (20°C)	
Hydrogen	1284	Glycerine	1904	Iron	5960
Carbon Dioxide	259	Sea Water	1535	Pyrex Glass	5640
Oxygen	316	Water	1493	Aluminum	5100
Nitrogen	350	Mercury	1450	Lead	2160
Air	344	Methyl Alcohol	1103	Rubber	1550

For Your information:

- Sound is produced by vibrating objects.
- Sound waves are longitudinal waves
- Sound has properties of all other waves: reflection, refraction, interference, diffraction

Speed of sound in air

- **Newton's Formula:** Newton proved that when longitudinal waves travel in an elastic medium, the velocity is given by $v = \sqrt{\frac{E}{\rho}}$ Where E is the modulus of elasticity of the medium and ρ is its density

Note: Wave velocity in a medium is fixed. Wave velocity is a material constant. It does not depend on wavelength, frequency and intensity

- **For Solids:** Modulus of elasticity

E = Young's modulus of elasticity = Y

$$v = \sqrt{\frac{Y}{\rho}}$$

- **For liquids:** Modulus of elasticity

E = Bulk modulus of elasticity = B

$$v = \sqrt{\frac{B}{\rho}}$$

- **For gases:** For a gaseous medium. Newton assumed that the propagation of longitudinal wave is an isothermal process (temperature remains constant) in this case modulus of elasticity

E = Pressure of the gas = P

$$v = \sqrt{\frac{P}{\rho}}$$

Note:

- The experimental results did not confirm to Newton's assumption. Laplace corrected the formula by arguing that sound waves travel adiabatically. Hence,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma KT}{m}}$$

where, T is absolute temperature of the gas, M is its molecular weight and R is universal gas constant.

$k = \frac{R}{N}$ = Boltzmann constant, N = Avogadro number

$m = \frac{M}{N}$ = Mass of one molecule

- Velocity of longitudinal wave in: **solid medium > liquid medium > gaseous medium**

Effects on the speed of sound in a Gas

- **Effect of pressure:** With the change of pressure, the velocity of sound in a gas remains unchanged, that is, there is no effect of pressure on the velocity of sound in gas.

CRITICAL CONCEPT

When explosion due to fusion reactions take place on the surface of sun then why we cannot hear their sound.

Effect of temperature: Velocity of sound $\propto \sqrt{T}$. Thus, the velocity of sound is directly proportional to the square root of absolute temperature (T).

$$v_t = v_0 \left(\frac{T}{T_0} \right)^{1/2}$$

$$v_t = v_0 + 0.61 t$$

Effect of moisture: The presence of moisture in the air reduces the resultant density of air. The conclusion is that the speed of sound increases with humidity. Hence the velocity of sound in damp air is greater than its value in dry air.

Effect of density: The speed of sound in a gas varies inversely as the square of the density of the gas. At the same temperature and pressure of gases:

Effect of Wind

If the air carrying sound waves, is itself moving i.e. there is wind. The speed of sound in the direction of wind relative to the ground is $(v + v_w)$ while against the wind is $(v - v_w)$, where v_w is the speed of wind and v is the speed of sound.

INTERESTING INFORMATION



For your Information:

- The speed of sound is higher in liquids and solids than it is in gases.
- The speed of sound in air increases 0.6 ms^{-1} for each 1°C increases.
- If the speed of a body in air exceeds the speed of sound, then it is called supersonic. Such a body leaves behind it a conical region of disturbance which spread continuously. Such a disturbance is called "shock wave". These waves may make cracks in window panels.

CRITICAL THINKING?

- The speed of sound in air is a function of
A. Wavelength B. Frequency
C. Temperature D. Amplitude
- The velocity of sound in air would become double than its velocity at 0°C at temperature
A. 313°C B. 586°C
C. 819°C D. 1172°C

PRINCIPLE OF SUPERPOSITION / SUPERPOSITION OF SOUND WAVES

If two or more waves propagate simultaneously in a medium, then the resultant displacement is given by the vector sum of displacement due to individual waves.

If the displacement given by the various waves to the particle are $y_1 + y_2 + y_3 + \dots + y_n$, then the resultant displacement of the particle is $y = y_1 + y_2 + y_3 + \dots + y_n$.

Different phenomenon due to principle of superposition are

- (a) Interference (b) Beats (c) Stationary waves

Interference of Sound

Superposition (mixing up) of two identical sound waves while passing through same medium propagating along same direction is called their interference.

Conditions for interference

- (i) coherent waves (ii) same medium (iii) same direction
(iv) identical waves (v) Sources of sound should be close to each other

- In constructive interference, two interfering sound waves reinforce each other, so that the resultant is a louder sound.

Condition for Constructive Interference

Path difference = $n\lambda$ where $n = 0, +1, +2, \dots$

- In destructive interference, two interfering sound cancel each other's effect, so that the resultant loudness of sound wave is become fainter.

Condition for Destructive Interference

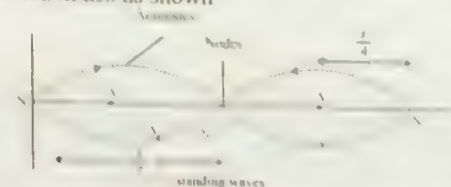
Path difference = $\left(n + \frac{1}{2}\right)\lambda$ where $n = 0, +1, +2, \dots$

- Echoing zone is region of constructive interference.
- Silence zone is region of destructive interference.
- Path difference is the difference between lengths of paths traveled by two waves in reaching the same point.

STATIONARY WAVES / STANDING WAVES

Stationary Waves

- Superposition of two identical waves traveling opposite to each other in the same medium simultaneously, gives rise to stationary or standing waves.
- Points of constructive interference are called *antinodes* while points of destructive interference are called *nodes* as shown.



- Amplitude is maximum at antinodes and minimum (zero) at nodes.
- Nodes are stationary points whereas antinodes are points that vibrate with maximum amplitude. Two consecutive nodes or antinodes are separated by distance equal to $\lambda/2$ and an antinode and its consecutive node by $\lambda/4$.

For your information:

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

Example: Progressive waves of frequency 300 Hz are superimposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart. What is the speed of the progressive waves?

- A) 100 m s⁻¹ B) 200 m s⁻¹ C) 450 m s⁻¹ D) 900 m s⁻¹

Solution: D) The distance 1.5 m corresponds to half of a wavelength, λ . The wavelength is thus given by $\lambda = 2(1.5) = 3.0$ m

The speed of the wave, $v = f\lambda = (300)(3) = 900$ m s⁻¹

STATIONARY WAVES IN A STRETCHED STRING/FUNDAMENTAL FREQUENCY AND HARMONICS

- The vibrations of a thin, long and perfectly elastic string are transverse stationary.
- On both the ends of string there are nodes, and an antinode is there in the middle.

- The speed of transverse wave in a stretched string is given by

$$v = \sqrt{\frac{T}{m}}$$

Where T and m are respectively the tension and mass per unit length of the string.

- Modes of vibration in a stretched string are as under

- If a string of length l vibrates in one loop then $\lambda_1 = 2l$

and frequency $f_1 = \frac{v}{2l} = \frac{1}{2l} \sqrt{\frac{T}{m}}$. This frequency is called

the fundamental note or first harmonic.

- If the string vibrates in two loops, then $\lambda = l$ and

$f_2 = \frac{v}{l} = 2f_1$. This frequency is called the first overtone or

second harmonic.

- If the string vibrates in three loops, then

$$\frac{3\lambda_3}{2} = l \rightarrow \lambda_3 = \frac{2l}{3}$$

$\therefore f_3 = \frac{3v}{2l} = 3f_1$. This frequency is called the second overtone or third harmonic.

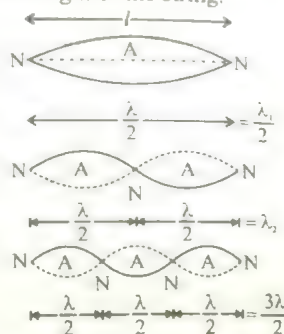
Both the odd and even harmonics are emitted from a stretched string. That is $f_n = nf_1$, where $n = 1, 2, 3, \dots$

Example: The frequency of the fundamental mode of transverse vibration of a stretched wire 100 mm long is 256 Hz. When the wire is shortened to 400 mm at the same tension, what is the fundamental frequency?

- A) 162 Hz B) 312 Hz C) 416 Hz D) 640 Hz

Solution: D) The frequency of the fundamental mode of a stretched wire is $f = \frac{1}{2l} \sqrt{\frac{T}{m}}$

$$f \propto \frac{1}{l} \Rightarrow \frac{f_2}{f_1} = \frac{l_1}{l_2} \Rightarrow \frac{f_2}{256} = \frac{100}{400} \Rightarrow f = 640 \text{ Hz}$$



STATIONARY WAVES IN AIR COLUMNS

An organ pipe is a pipe that sets in vibration the air enclosed in it when the air is blown into it. As a result, sound is produced in it.

Organ pipes are of two types – closed end organ pipe and open end organ pipe

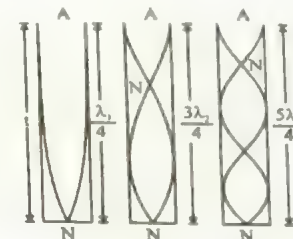
An open end organ pipe has both its ends open.

A closed end organ pipe has one of its ends closed and the other open

In a closed end pipe a node is always formed at the closed end and an antinode is formed at the open end.

Longitudinal stationary waves are formed in an organ pipe.

Various stages of resonance in a **CLOSED END** organ pipe are represented in the following diagrams.



- If the length of the pipe $l = \frac{\lambda_1}{4}$ then $\lambda_1 = 4l$, \therefore Frequency, $f_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$

This frequency is called fundamental frequency or fundamental note or first harmonic

- If $l = \frac{3\lambda_2}{4}$ then $\lambda_2 = \frac{4l}{3}$, \therefore Frequency, $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l} = 3f_1$

This frequency is called third harmonic or first overtone.

- If $l = \frac{5\lambda_3}{4}$ then $\lambda_3 = \frac{4l}{5}$, \therefore Frequency, $f_3 = \frac{v}{\lambda_3} = \frac{5v}{4l} = 5f_1$

This frequency is called fifth harmonic or second overtone.

- Only odd harmonics can be produced in a closed end organ pipe. That is

$$f_n = (2n-1)f_1 \text{ where } n = 1, 3, 5, \dots$$

- Longitudinal stationary waves are formed in an **OPEN END ORGAN PIPE** too

- The antinodes are formed at both the ends of an open pipe.

- Various stages of resonance in an open organ pipe have been represented in the following diagrams

CRITICAL CONCEPT

Under what principle a sound is produced in a flute?



$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \quad f_2 = \frac{v}{\lambda_2} = \frac{v}{L} \quad f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$$

This frequency is called the fundamental frequency or the first harmonic.

$$\text{If } f = \lambda_2 \text{ then } f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f$$

This frequency is called the second harmonic or first overtone.

$$\text{If } f = \frac{3\lambda_3}{2} \text{ then } \lambda_3 = \frac{2L}{3} \therefore \text{Frequency, } f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f$$

This frequency is called the third harmonic. Three harmonics are produced in an open-end organ pipe. That is $f_n = nf$, where $n = 1, 2, 3$.

The sound emitted by an open-end organ pipe is musical.

No. of harmonics in open pipe = 2 x No. of harmonics in closed pipe

Example: 2nd overtone of an open organ pipe resonates with 3rd harmonics of a closed organ pipe. The ratio of their length will be

- A) $\frac{2}{1}$ B) $\frac{1}{2}$ C) $\frac{6}{5}$ D) $\frac{5}{6}$

Solution: A) $\frac{L_1}{L_2} = \frac{3\lambda_2 \cdot 2}{3\lambda_1 \cdot 4} = \frac{2}{1}$

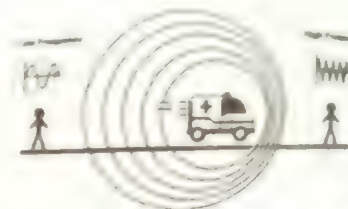
CRITICAL THINKING?

- The ratio of the fundamental frequency of an open ended pipe to a pipe whose one end is closed is
A) 1:2 B) 2:1
C) 1:1 D) 1:4
- Which one is the correct relation for fundamental frequency of open and closed pipe?
A) $f_{\text{open}} = 2f_{\text{closed}}$ B) $f_{\text{closed}} = 2f_{\text{open}}$
C) $f_{\text{open}} = f_{\text{closed}}$ D) $f_{\text{open}} = 1/f_{\text{closed}}$

DOPPLER'S EFFECT

Doppler Effect (Frequency Shift)

- Apparent change in pitch (frequency) of sound due to relative motion of source and observer.
- Doppler's effect was discovered by Doppler, an Australian physicist, in 1845.
- Apparent frequency of sound heard by stationary listener due to source moving towards him at speed 'u' is given as,



$$f' = \left(\frac{v}{v-u} \right) f \quad f' > f \text{ or } \lambda' = \left(\frac{v-u}{v} \right) \lambda \quad \lambda' < \lambda$$

- Apparent frequency of sound heard by stationary listener due to source moving away from him at speed 'u' is given as,

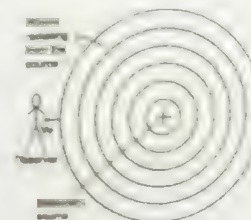
$$f' = \left(\frac{v}{v+u} \right) f \quad f' < f \text{ or } \lambda' = \left(\frac{v+u}{v} \right) \lambda \quad \lambda' > \lambda$$

- Apparent frequency of sound heard by a person moving towards a stationary source with speed 'u' is given as:



$$f' = \left(\frac{v+u}{v} \right) f \quad f' > f \text{ or } \lambda' = \left(\frac{v}{v+u} \right) \lambda \quad \lambda' < \lambda$$

- Apparent frequency of sound heard by a listener moving away from a stationary source with speed 'u' is given as:



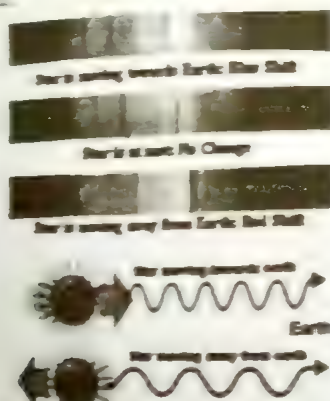
What will sound be perceived more (louder or softer)?

When the speed of sound is = speed of observer = speed of source
then both source and observer move away from each other

$$f' = \frac{v}{v + v_s} f$$

Applications of Doppler's effect

- (i) Ships and submarine (sonar devices)
- (ii) Bats for travelling
- (iii) Radar devices
- (iv) Determining velocity of a star or a galaxy
- (v) To measure blood flow in major arteries
- (vi) When a star is moving away from Earth then wavelength of light increases and red shift is observed (red shift)
- (vii) When a star is moving towards the Earth then wavelength of light decreases and blue shift is observed.



Example: A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed of sound = 330 m/s)

A. 470

C. 517

B. 490

D. 540

Solution: D

$$f' = \frac{v}{v - v_s} f = 450 \times \frac{330}{330 - 33} = 540 \text{ Hz}$$

CRITICAL CONCEPT

Can you apply Doppler's effect for light wave and source of light?

CRITICAL THINKING?

6. A car is traveling at 20 m/s away from a stationary observer. If the car's horn emits a frequency of 600 Hz, what frequency will the observer hear? (Use $v = 340$ m/s for the speed of sound.)

A. $(34 - 36) \times 600$ HzB. $(34 + 32) \times 600$ HzC. $(34 - 24) \times 600$ HzD. $(32 + 34) \times 600$ Hz

SIMPLE HARMONIC MOTION

- A body's motion is said to be simple harmonic if it moves to and fro about a fixed position along same path. e.g.
 - (i) Motion of simple pendulum, (ii) Motion of molecules in a solid
- Simple harmonic motion (SHM) is a special type of vibratory motion in which
 - (i) $a \propto -x$ (ii) a is directed towards mean position.
- Restoring force is always directed towards mean position hence assigned negative sign.

Example: A simple harmonic oscillator has a time period of 10 seconds. Which equation relates its acceleration a and displacement x ?

A. $a = -10x$ B. $a = -(20\pi)x$ C. $a = -(20\pi)^2 x$ D. $a = -(2\pi)^2 x$

Solution: D. $a = -\omega^2 x$ and $\omega = \frac{2\pi}{T}$

Where T is the period.

$$a = -\left(\frac{2\pi}{10}\right)^2 x$$

CRITICAL THINKING?

7. What is constant in S.H.M?

A. Restoring force

B. Kinetic energy

C. PE

D. Periodic time

CHARACTERISTICS OF SIMPLE HARMONIC MOTION

- Instantaneous displacement is distance covered by body at any instant from mean position.
- Periodic motion is that which repeats itself after equal time intervals.
- Vibration is one complete round trip of a body about its mean position.
- Time period is defined as time taken by vibrating body to complete its one round trip denoted by T .

CRITICAL CONCEPT

Can a linear motion of a body be SHM?

- Frequency is number of vibrations per second and denoted by f . Its unit is Hz. Other units are vibrations/s, cycle/s, rev/sec.
- Amplitude is maximum distance from mean position.
- Angular frequency is $\omega = 2\pi f \Rightarrow \omega = 2\pi$

SIMPLE PENDULUM

It consists of a heavy point mass suspended from a rigid support by means of almost weightless and inextensible string.



- Galileo invented simple pendulum.
- Motion of simple pendulum is S.H.M if there is no damping.
- Damping force reduces the amplitude of simple pendulum continuously and finally its motion is stopped.
- In absence of damping force, restoring force on simple pendulum is given as, $F_r = -mg \sin\theta$, and for small amplitude oscillations $F_r = -mg\theta$.
- Equation of acceleration of simple pendulum for small amplitude is: $a = -\left(\frac{g}{l}\right)x$

Thus $a = -\frac{g}{l}x$ for simple pendulum and does not depend on mass like the mass-spring system does.

- Time period and frequency of simple pendulum are given as;

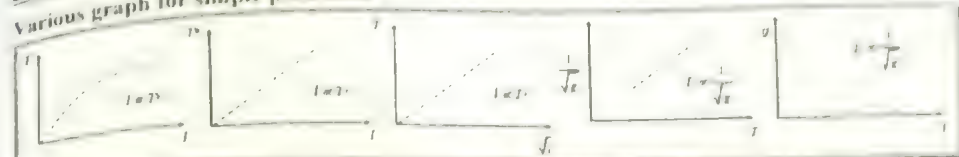
$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{and} \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

- If amplitude of simple pendulum is not small then, it has non-S.H.M as $a = -g \sin\theta$ and we know that $\sin\theta = \theta$ only when θ is small.
- A second pendulum has following characteristics;

Time period	2 seconds
Frequency	0.5 Hz
Length	0.99 cm or 1 meter

CRITICAL CONCEPT

Every vibrating body produces a sound. Does a simple pendulum also produce a sound?

Various graph for simple pendulum

Example: The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
 A) first decrease and then increase to the original value
 B) first increase and then decrease to the original value
 C) increase towards a saturation value
 D) remain unchanged

Solution: B) Centre of mass of combination of liquid and hollow portion (at position l), first goes down (to $l + \Delta l$) and when total water is drained out, center of mass regain its original position (to l)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T first increases and then decreases to original value

For your Information:

If a pendulum is shifted from Karachi to Quetta, then its time period will be increased

ENERGY CONSERVATION IN SHM**Energy Conservation in SHM**

- Its K.E is given as;

$$K.E_{\text{ins}} = \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2}\right)$$

$$(K.E)_{\text{max}} = \frac{1}{2} k x_0^2$$

It is at mean position.

$$(K.E)_{\text{min}} = 0$$

It is at extreme position.

$$K.E_{\text{ins}} = (K.E)_{\text{max}} \left(1 - \frac{x^2}{x_0^2}\right)$$

- Its P.E is given as;

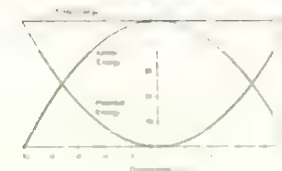
$$P.E_{\text{ins}} = \frac{1}{2} k x^2$$

$$(P.E)_{\text{max}} = \frac{1}{2} k x_0^2$$

It is at extreme position.

$$(P.E)_{\text{min}} = 0$$

It is at mean position.



Frequency of K.E. and P.E. is twice of frequency of S.H.M. but frequency of T.E. is same.

Example: A mass of 8.0 g oscillates in simple harmonic motion with an amplitude of 5.0 mm at a frequency of 40 Hz. What is the total energy of this simple harmonic motion?

A) 0.16 mJ

C) 6.3 mJ

B) 1.6 mJ

D) 640 mJ

Solution: $E = \frac{1}{2} m \omega^2 x^2$

$$= \frac{1}{2} (8 \times 10^{-3}) (2\pi \times 40)^2 (5.0 \times 10^{-3})^2$$

$$= \frac{1}{2} (8 \times 10^{-3}) (2\pi \times 40)^2 (5.0 \times 10^{-3})^2 = 0.16 \text{ mJ}$$

CRITICAL THINKING?

8. A body performs S.H.M. Its kinetic energy K varies with time t as indicated by graph



TOPIC-5 THERMODYNAMICS

COURSE CONTENT

- Thermal Energy is transferred from the region of higher temperature to the region of lower temperature
- First law of thermodynamics
- Specific heat and molar specific heat, specific heat capacity
- Relation $C_p - C_v = R$
- Internal Energy
- Heat and Work

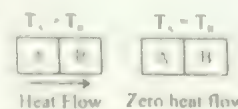
TEMPERATURE

Temperature is sometimes called the degree of hotness or coldness of a body and is a quantity which is such that when two bodies are placed in contact, heat flows from the body at the higher temperature to the one at lower temperature

HEAT

- Heat is the energy which is transferred due to difference in temperature of the bodies.
- It is a form of energy
- The temperature of a system increases if heat is supplied to or absorbed by it.
- Temperature of system decreases when heat is taken out of the system
- Often, if the temperature changes by an exchange of heat, then $Q = mc(\Delta T)$ where m = mass, c = specific heat, $T_2 - T_1 = \Delta T$ = change in temperature
- The unit of heat in MKS is joule, in CGS, it is erg and in thermal units calorie.
- 4.2 Joule = 1 calorie

Thermal Equilibrium: Objects A and B are in contact. If heat flows from A to B, then A is at a higher temperature than B. When the heat flow from A to B is zero, the two objects are in thermal equilibrium.



Zeroth Law: When two systems A and B are separately in thermal equilibrium with a third system C, then the first two systems will also be in thermal equilibrium with each other. It means if,

$$T_A = T_C \quad \text{and} \quad T_B = T_C \quad \text{then,} \quad T_A = T_B$$



Thus according to this law, temperature is that intrinsic property of an object on the basis of which, we can say that whether the object is in thermal equilibrium with another or not.

For Your Information:

- A thermodynamics system which is surrounded by a distinct boundary



- An open system which transfer both energy and matter
- A closed system which transfers only energy
- An isolated system which transfers neither energy nor matter.

CRITICAL THINKING?

1. Normal temperature of a human body is 98.6°F while its atmosphere temperature is 84.6°F . What will be the temperature of the dead body in such atmosphere

A. 84.6°F
C. 92.5°F

B. 98.6°F
D. 185°F

FIRST LAW OF THERMODYNAMICS

When heat is transformed into other forms of energy total heat remains constant
 $\Delta Q = \Delta U + \Delta W$ where ΔQ is positive when heat is added and vice versa.
 ΔW is positive when work is done by system and vice versa.

Inferences from 1st Law of Thermodynamics: $\Delta U = \Delta Q - \Delta W$

$$\left(\begin{array}{c} \text{change in internal} \\ \text{energy} \end{array} \right) = \left(\begin{array}{c} \text{Heat energy flowing in} \\ \text{as mechanical work} \end{array} \right) - \left(\begin{array}{c} \text{Heat energy flowing out} \\ \text{as mechanical work} \end{array} \right)$$

- Internal energy is a state function. i.e. depends on initial and final states
- For a cyclic process, we have: $\Delta U = 0$, $U_i = U_f$ Then $\Delta Q = \Delta W$

ISOTHERMAL PROCESS, ADIABATIC PROCESS, ISOBARIC PROCESS & ISOCHORIC PROCESS



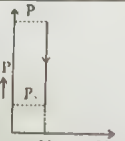
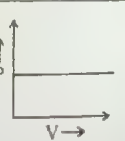
Isothermal process is that in which temperature remains constant.

$$\Delta Q = \Delta W \text{ as } \Delta U = 0$$

- Isochoric process is that in which volume remains constant.
 $Q = \Delta U$ as $\Delta W = 0$
- Isobaric process is that in which pressure remains constant. $\Delta Q = \Delta U + P\Delta V$
- Adiabatic process is that in which no heat enters or leaves the system such that temperature remain constants

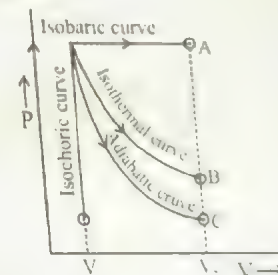
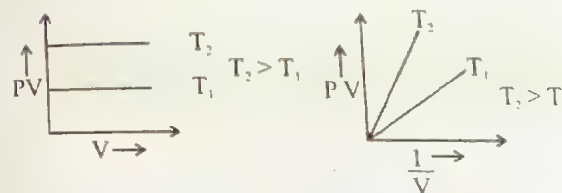
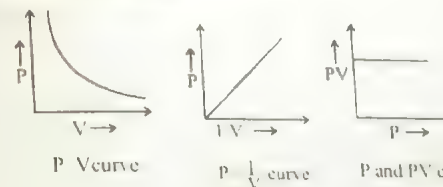
Note: Cooling is produced when adiabatic expansion takes place and heating is produced during adiabatic compression.

COMPARISON OF DIFFERENT THERMODYNAMICAL PROCESSES

Sr.	Property	Isothermal	Adiabatic	Isometric (Isochoric)	Isobaric
1.	Condition	$T = \text{constant}, \Delta T = 0, \Delta U = 0$	$Q = \text{constant}, \Delta Q = 0$	$V = \text{constant}, \Delta V = 0, \Delta W = 0$	$P = \text{constant}, \Delta P = 0$
2.	Form of first Law	$\Delta Q = 0 + \Delta W = P\Delta V$	$0 = \Delta U + \Delta W, \Delta W = -\Delta U$	$\Delta Q = \Delta U + 0, \Delta U = n C_v \Delta T$	$\Delta Q = \Delta U + \Delta W = n C_p \Delta T$
3.	P.V Diagram				
4.	Equation of state	$PV = \text{constant}$	$PV^\gamma = \text{constant}$	$\frac{P}{T} = \text{constant}$	$\frac{V}{T} = \text{constant}$

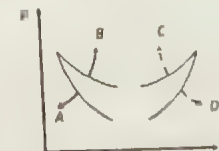
5.	Specific Heat	$c = \infty$	$c = 0$	$c = C_v$	$c = C_p$
6.	Slope of P.V curve	$\frac{\Delta P}{\Delta V} = -\frac{P}{V}$	$\frac{\Delta P}{\Delta V} = \gamma \frac{P}{V}$	$\frac{\Delta P}{\Delta V} = \infty$	$\frac{\Delta P}{\Delta V} = 0$
7.	Example	(i) Isothermal expansion of ideal gas, (ii) Conversion of ice at 0°C to water of 0°C	(i) Burst of air tube (ii) Propagation of sound in air (iii) Refrigeration	(i) To supply heat at constant volume, (ii) Atmospheric changes, (iii) Explosion in gases	(i) Melting of ice, (ii) Boiling of water

Different Thermodynamic Process and Relative: Thermodynamics Process



CRITICAL THINKING?

2. Four curves A, B, C and D are drawn in the adjoining figure for a given amount of gas. The curves which represent adiabatic and isothermal changes are.



A. C and D respectively
C. D and C respectively

B. A and B respectively
D. B and A respectively

SPECIFIC HEAT AND MOLAR SPECIFIC HEAT/SPECIFIC HEAT CAPACITY.

- Amount of heat required to raise the temperature of a substance through 1 K is called *heat capacity*, denoted by C .
 $Q = C \Delta T$
- Specific heat* is the amount of heat required to raise the temperature of unit mass through unit temperature.
 $Q = C \Delta T$

Molar Specific heat at Constant Volume

- Molar specific heat at constant volume is the amount of heat required to raise the temperature of one mole of the gas through 1K keeping volume constant.

Specific Heat at Constant Pressure

- Molar specific heat at constant pressure is the amount of heat required to raised the temperature of one mole of the gas through 1K keeping pressure constant.

Relation $C_p - C_v = R$:

- $\Delta Q_v = n C_v \Delta T$ (Heat supplied at constant volume).
- $\Delta Q_p = n C_p \Delta T$ (Heat supplied at constant pressure).
- $C_p - C_v = R$.

$$\frac{C_p}{C_v} = \gamma$$

Specific Heat The heat required to increase the temperature of unit mass of substance by 1°C is called the specific heat of the substance. $Q = mc\Delta T$ or $c = \frac{Q}{m\Delta T}$.

TYPES OF GAS	C_v	C_p	$\gamma = \frac{C_p}{C_v}$
Monoatomic	$\frac{3}{2}R$	$\frac{5}{2}R$	1.67
Diatomic	$\frac{5}{2}R$	$\frac{7}{2}R$	1.40
Polyatomic	$3R$	$4R$	1.33

Example: In an experiment to determine the specific heat capacity of a liquid by an electrical method, a student obtained the following results.

Mass of liquid heated	1.5 kg
Initial liquid temperature	300 K
Final liquid temperature	357 K
Electrical power of heater	1.0 kW
Time of heating	180 s

What is the specific heat capacity of the liquid?

- A) $2.1 \text{ J kg}^{-1} \text{ K}^{-1}$ C) $1800 \text{ J kg}^{-1} \text{ K}^{-1}$
 B) $18 \text{ J kg}^{-1} \text{ K}^{-1}$ D) $2100 \text{ J kg}^{-1} \text{ K}^{-1}$

Solution: D) The specific heat capacity, $c = \frac{Q}{m\Delta T}$

$$= \frac{\text{Power} \times \text{time}}{\text{mass} \times \Delta T} = \frac{(1.0 \times 10^3)(180)}{(1.5)(357 - 300)} = 2100 \text{ J kg}^{-1} \text{ K}^{-1}$$

CRITICAL THINKING?

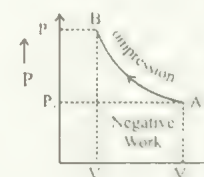
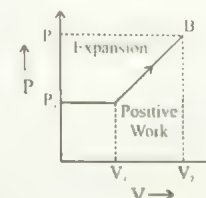
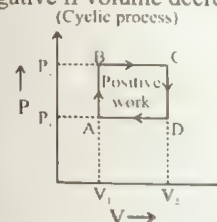
3. Out of following whose specific heat is maximum?
 A. lead B. Brass
 C. Glass D. Iron

INTERNAL ENERGY**Internal Energy**

- The energy possessed by a system due to molecular motion and configuration is considered as its internal energy.
- Internal energy of a system = Kinetic energy of its constituents molecules + Potential energy of its constituents molecules.
- Internal energy is a state function which depends on pressure, temperature, volume etc. of the system.
- The change in the internal energy of a system depends only on initial and final states of the system and not on the path followed.
- For an ideal gas the potential energy of the molecules is zero. Here internal energy = Kinetic energy of molecules and it depends on temperature.

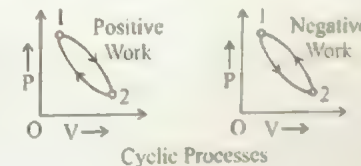
HEAT AND WORK

- "Heat is a form of energy which flows from the hotter body to the colder body till the temperatures of the two bodies become equal".
- Work can be defined as the energy that is transferred one body to the other owing to a force that acts between them. The amount of work done by a system as it expands or contracts is given by: $W = P\Delta V$
- Work is taken to be positive if the system expands against some external force. Work is taken to be negative if the system contracts because of some external force exerted by the surroundings.
- Work calculation by indicator-diagram method:** It is positive if volume increases and negative if volume decreases.



- If area under PV-diagram is traced in clockwise direction the work done will be positive (expansion) and will be negative (during compression) if the area is traced in anticlockwise direction.

- Cyclic Process:** A cyclic process is one in which the thermodynamic variables periodically return to their original values. In other words, the initial and final states of the system coincide. The work done by a cyclic process is equal to the area enclosed by the loop.



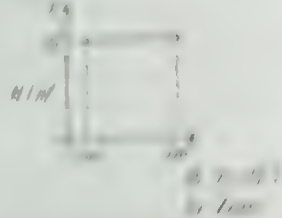
- Potential work: If the system is moved in the electric field...
- Potential work: If the system is moved in the electric field...

If a system moves in a potential field V from V_1 to V_2 then the work done is given by
 $W = -q(V_2 - V_1)$ The change in potential energy of a system is given by $\Delta U = q(V_2 - V_1)$



CRITICAL THINKING?

4. The work done in fig.



TOPIC 6

ELECTROSTATICS

CONCEPTS

- Electric field and its intensity
- Application of Coulomb's law to calculate force between two point charges
- Electric field lines, their properties and applications
- Electric potential
- Calculation of potential at a point due to a point charge
- Calculation of potential of a system of point charges
- Electric field and potential of a dipole
- Electric field and potential of a uniformly charged rod

Coulomb's Law is an inverse square law

It states that the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Vector form of Coulomb's Law

Vector form of Coulomb's law is

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Where \hat{r}_{12} is the unit vector from charge 1 to charge 2.

Note: Coulomb's law depends upon system of units and medium between the charges.

(i) Effect of Units:

- In SI system for $k = 1$, $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$
- In CGS system for $k = 1$, $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

(ii) Coulomb's Law in Material Media

When a dielectric medium is completely filled in between charges, the force between the charges is reduced by a factor of ϵ_r (relative permittivity) compared to the force in vacuum.

$$F_m = \frac{F_v}{\epsilon_r} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2}$$

CRITICAL CONCEPT

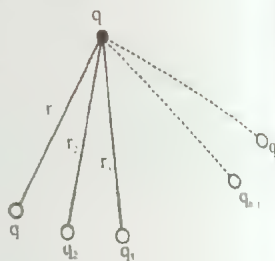
- Does an electrostatic force exist between a charged and an uncharged bodies?
- If an object transfers its charges to another object, will its mass be affected?
- If 2×10^9 electrons are transferred from a glass rod to a piece of silk by rubbing, what will be the net charges on both objects?

Principle of Superposition

According to the principle of superposition, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges. Consider number of charges $q_1, q_2, q_3, \dots, q_n$ are applying force on a charge q

Net force on q will be

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_{n-1} + \vec{F}_n$$



For Your information:

- A Charge is transferred from one body to another by electrons.
- Like energy and momentum, charge is also a conserved and quantized quantity.
- Both electrostatic and gravitational forces between masses obey inverse square law.

CRITICAL THINKING?

- The number of electrons or protons which constitutes a charge of one coulomb is
A. 6.25×10^{-18} B. 6.25×10^{18}
C. 1.6×10^{-19} D. 1.6×10^{19}
- If the distance between two equal charges is reduced to half and the magnitude of charges is also decreased to half, then the force between them will be
A. Remain same B. Decreased to half
C. Increased to double D. Becomes four time

ELECTRIC FIELD AND ITS INTENSITY

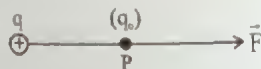
Electric Field

Electric field due to a point charge is the space surrounding it, within which electric force can be experienced by the another charge.

Electric field intensity (\vec{E})

The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point.

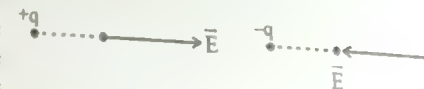
$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2} \hat{r}$$



Unit: $\text{NC}^{-1}(\text{S.I})$, Vm^{-1} and $\text{dyne/stat-C}(\text{CGS})$

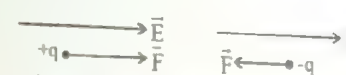
Direction of electric field

Electric field (intensity) \vec{E} is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge.



Relation between electric force and electric field

In an electric field \vec{E} a charge (q) experiences a force $F = qE$. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field.



The electric field of a continuous charge distribution at some point.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

CRITICAL CONCEPT

Does an electric field exist in empty space?

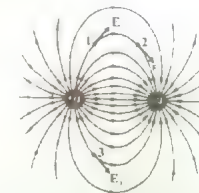
Where q is the charge on one element of the charge distribution r is the distance from the element to the point under consideration. \hat{r} is the unit vector directed from the position of eliminated charge towards the point where electric field is to be found out?

- The path followed by a tiny positive charge in an electric field is called line of force.

- Electric lines of force are imaginary lines starting from positive charge and ending on negative charge.

Some of important properties are given below;

- Originate from positive charge.
- End on negative charge.
- Do not intersect
- Contract longitudinally.
- Repel transversely.
- No electric line is present inside the conductor.
- Tangent drawn to electric lines gives the direction of electric intensity
- Electric field is stronger where the electric lines are closely packed



Equipotential Surface

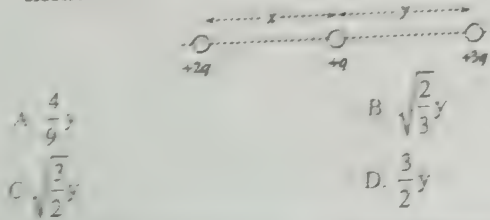
- If every point of a surface is at same potential, then it is said to be an equipotential surface.
- The direction of electric field is perpendicular to the equipotential surfaces or lines of force.
- The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.
- Equipotential surfaces can never cross each other.
- Work done in moving any charge over these surfaces is zero.

For Your information:

- There is no electric field inside the conductor.

CRITICAL THINKING?

3. A metallic charged sphere is placed in uniform electric field E , the electric field inside the sphere will be
 A. E
 B. Less than E
 C. Greater than E
 D. Zero
4. The figure below shows three point charges, all positive. If the net electric force on the center charge is zero, what is the value of $\frac{y}{x}$?



A. $\frac{4}{9}y$
 C. $\frac{2}{\sqrt{2}}y$

B. $\sqrt{\frac{2}{3}}y$
 D. $\frac{3}{2}y$

GRADIENT OF $E = \frac{\Delta V}{\Delta d}$ TO CALCULATE THE FIELD STRENGTH

We can relate electric potential difference and electric field intensity by following relation:

$$E = -\frac{\Delta V}{\Delta r}$$

Where negative shows that \vec{E} is along decreasing potential.

- (i) We can call 'E' potential gradient because it represents the maximum rate of change of potential difference w.r.t displacement.

- (ii) SI unit of E (N/C) is equivalent to V/m . $\frac{1N}{1C} = \frac{1V}{1m}$

Example: A thunder-cloud whose base is 500 m above the ground. The potential difference between the base of the cloud and the ground is 200 MV. A raindrop with a charge of $4.0 \times 10^{-6} C$ is in the region between the cloud and the ground. What is the electrical force on the raindrop?

- A) $1.6 \times 10^{-3} N$ B) $8.0 \times 10^{-3} N$ C) $1.6 \times 10^{-3} N$ D) $0.40 N$

Solution: A) $F = qE = q \left(\frac{V}{r} \right) = (4.0 \times 10^{-6}) \left(\frac{200 \times 10^6}{500} \right) = 1.6 \times 10^{-3} N$

GRADIENT OF $E = \frac{Q}{4\pi\epsilon_0 r^2}$ FOR THE FIELD STRENGTH

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$E \propto \frac{1}{r^2} \text{ (Inverse square law)} \Rightarrow \frac{E_1}{E_2} = \left(\frac{r_2}{r_1} \right)^2$$

Graph:



Particulars

GRAVITATIONAL FORCE AND ELECTRIC FORCE

Formula	Gravitational force	Electric force
Range	Infinite	Infinite
Symbol of constant	G	k
Value of constant	Very small	Very large
Nature	Always attractive	Both repulsive and attractive
Dependence	Medium independent	Medium dependent
Relative strength	Weak, can be felt with mass	Strong, can be felt with mass

APPLICATION OF GAUSS'S LAW (ELECTRIC FIELD INTENSITY DUE TO AN INFINITE SHEET OF CHARGE, ELECTRIC FIELD INTENSITY BETWEEN TWO OPPOSITELY CHARGED PARALLEL PLATES)

- Electric intensity due to infinite sheet of charge is given as $E = \frac{\sigma}{2\epsilon_0}$
- Electric intensity between two equal but oppositely charged plates is given as $E = \frac{\sigma}{\epsilon_0}$

ELECTRIC POTENTIAL

Electric potential difference between two points is defined as

"Work done per unit positive charge in moving it against electric field with uniform velocity, i.e. keeping the charge in electrostatic equilibrium."

$$\Delta V = \frac{W_{A \rightarrow B}}{q}$$

- It can be given in terms of potential energy as

$$\Delta V = \frac{\Delta U}{q} \quad \therefore W_{A \rightarrow B} = \Delta U = \text{electrostatic P.E.}$$

- SI unit of potential difference is volt.

"If 1J of work is done on one coulomb charge between two points keeping it in equilibrium, the potential difference is 1V"

$$1V = 1J/1C$$

- Absolute electric potential at a distance 'r' from source is given as

$$V_{\text{abs}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

CRITICAL CONCEPT

- What will be the value of electric potential when a positive charged particle moves in the direction of electric field?
- What will be the work done on the charged particle when it is displaced between two points which have same potential?

Topic-6

Example: A conducting hollow sphere of radius 0.1 m is given a charge of $10 \mu\text{C}$. The electric potential on the surface of sphere will be
 A) zero B) $3 \times 10^5 \text{ V}$ C) $9 \times 10^5 \text{ V}$ D) $9 \times 10^9 \text{ V}$

Solution: C) $V_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = \frac{9 \times 10^9 \times 10 \times 10^{-6}}{10^{-1}} = 9 \times 10^5 \text{ V}$

- **Absolute potential** at a point due to collection of point charges is given as;

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$
- ECG records the voltage between points on human skin generated by electrical process in the heart while EEG records that by brain.

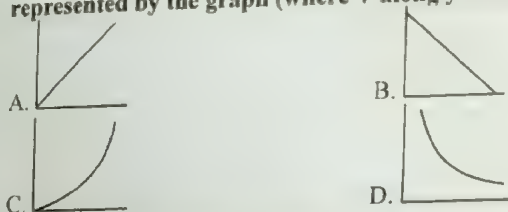
Electron Volt

The amount of energy acquired or lost by an electron as it traverses a potential difference of one volt.

- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ and $1 \text{ J} = 6.25 \times 10^{18} \text{ eV}$
 It is the unit of energy specially used for atomic particles

CRITICAL THINKING?

5. The variation of electric potential due to a point charge with distance is represented by the graph (where V along y-axis and r along x-axis)

**CAPACITOR (CAPACITANCE OF A CAPACITOR AND ITS UNIT)**

Capacitor is a device used for **storing electric charge** and electrical energy.

- Charge stored by capacitor is given as; $Q = CV$ Where C is **capacitance** of capacitor
- **Capacitance is defined as** Ability of a capacitor to store charge. OR
 "The ratio of charge stored to the potential difference between plates of capacitor."

Mathematically, $C = \frac{Q}{V}$

- SI unit of capacitance is **Farad**.
- **1 Farad** It is defined as;
 The capacitance of a capacitor is one farad if a charge of one coulomb, given to one of the plates of a parallel plate capacitor, produces a potential difference of one volt between them **1F = 1C/V**

Topic-6

CAPACITANCE OF PARALLEL PLATE CAPACITOR

Capacitance of parallel plate capacitor with air between plates

$$C_{\text{vac}} = \epsilon_0 \frac{A}{d}$$

The above expression shows that-

- As we increase the surface area of plate the capacitance will increase
- Decreasing the distance between plates will increase the capacitance
- Introducing a dielectric between the plates will increase the capacitance
- Capacitance of a parallel plate capacitor with dielectric between plates

$$C_{\text{med}} = \epsilon_r \epsilon_0 \frac{A}{d} = \epsilon_r C_{\text{vac}} \quad \text{so} \quad C_{\text{med}} > C_{\text{vac}}$$

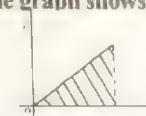
Dielectric co-efficient or Dielectric constant

Definition The ratio of the capacitance of a parallel plate capacitor with air as medium between the plates to its capacitance with vacuum (or air) as medium

them. $\epsilon_r = \frac{C_{\text{med}}}{C_{\text{vac}}}$

CRITICAL THINKING?

6. In the presence of dielectric material, the electric field between the plates of the capacitor will be
 A. Zero B. Remain same
 C. Decrease D. Increase
7. The graph shows the growth of charge with potential difference between plates. The area under the graph shows



- A. Capacitance
 C. Energy stored
 B. Separation of plates
 D. Electric intensity

SEPARATION IS INCREASING

	Battery is removed	Battery remains
Quantity		
Capacity	Decreases because $C \propto \frac{1}{d}$ i.e., $C_2 < C_1$	Decreases because $C \propto \frac{1}{d}$ i.e., $C_2 < C_1$

Charge	Potential difference	Electric field	Energy
Remains constant because a battery is not present i.e., $q_1 = q_2$	Increases because $V = \frac{q}{C}$ $\Rightarrow V \propto \frac{1}{C}$ i.e., $V_2 > V_1$	Remains constant because $E = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$ i.e., $E_2 = E_1$	Decreases because $U = \frac{q^2}{2C}$ Here $C_2 > C_1$ and $q_1 = q_2$ i.e., $U_2 < U_1$
Decreases because battery is present i.e., $q_2 < q_1$. Remaining charge $(q_1 - q_2)$ goes back to the battery.	$V_2 = V_1$ (since battery maintains the potential difference)	Decrease because $E = \frac{q}{A\epsilon_0}$ and $q_2 < q_1 \Rightarrow E \propto q$ i.e., $E_2 < E_1$	Decreases because $U = \frac{1}{2} CV^2$ Here $C_2 < C_1$ but $V_2 = V_1$ i.e., $U_2 < U_1$

SEPARATION IS DECREASING

Quantity	Battery is removed	Battery remains connected
Capacity	Increases because $C \propto \frac{1}{d}$ i.e., $C_2 > C_1$	Increases i.e., $C_2 > C_1$
Charge	Remains constant because battery is not present i.e., $q_2 = q_1$	Increase because battery is present i.e., $q_2 > q_1$. Remaining charge ($q_2 - q_1$) supplied from the battery.
Potential difference	Decreases because $V = q/C$ $\Rightarrow V \propto \frac{1}{C}$ and $C_2 > C_1$ i.e., $V_2 < V_1$	$V_2 = V_1$ (since battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\epsilon} = \frac{q}{A\epsilon}$ and $V_2 = V_1$ i.e., $E_2 = E_1$	Increases because $E = \frac{q}{A\epsilon_0}$ And $q_2 > q_1$ i.e., $E_2 > E_1$
Energy	Decreases because $U = \frac{q^2}{2C}$ And in this case $q_2 = q_1$ and $C_2 > C_1$ i.e., $U_2 < U_1$	Increases because $U = \frac{1}{2} CV^2$ Here $C_2 > C_1$ but $V_2 = V_1$ i.e., $U_2 > U_1$

ENERGY STORED IN A CAPACITOR

- Charge on the plate of capacitor **possesses electrical potential energy** because of the work done to deposit the charge on the plates.

$$P = \frac{1}{2} qV$$

$$\text{Energy in capacitor} = \frac{1}{2} CV^2 = \frac{1}{2} \frac{q^2}{C}$$

- Energy is stored in electric field between the plates.
- $$\text{Energy} = \frac{1}{2} \left[\frac{A \epsilon_r \epsilon_0}{d} \right] [Ed]^2$$

$$\text{Energy Density} = \text{Energy/volume} = \frac{1}{2} C_1 C_0 E^2$$

Example: A capacitor of capacitance $4\ \mu\text{F}$ is charged to $80\ \text{V}$ and another capacitor of capacitance $6\ \mu\text{F}$ is charged to $30\ \text{V}$. when they are connected together, the energy lost by the $4\ \mu\text{F}$ capacitor is

A) $7.8\ \text{mJ}$

- A) 7.8 mJ
B) 4.6 mJ
C) 3.2 mJ
D) 2.5 mJ

Solution: A)

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = 50V$$

For 4 μF capacitor $E_i = \frac{1}{2} C_i V^2$, $E_r = \frac{1}{2} C_i V_c^2$

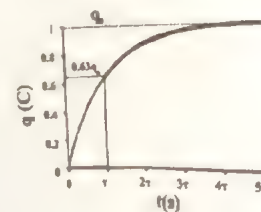
$$E_1 - E_f = \frac{1}{2} C (v^2 - v_c^2) = 7.8 \times 10^{-3} \text{ J}$$

CHARGING AND DISCHARGING A CAPACITOR

- D.C supply stores charges on the plate
- A.C supply does not store charge
- Charging and discharging time depends upon product of R & C
- **The relation for charging of a capacitor**

$$q = q_0 (1 - e^{-\frac{t}{RC}})$$

This relation shows the nature of charging of a capacitor is exponentially, where q_0 represents the maximum charge on the capacitor that stores after an infinite length of time. It means, the rate of charging of a capacitor is different at its different stages, graphically it is represented by



If time $t = 1 \tau$ (1 RC)

$$q = q_0 (1 - e^{-\frac{t}{\tau}})$$

$$q = q_o(1 - e^{-1}) \Rightarrow q = q_o(1 - \frac{1}{e})$$

$$q = q_0 \left(1 - \frac{1}{2.718} \right) \Rightarrow q = 0.63 q_0$$

This gives that after one time constant, the capacitor will be charged 63%.

Topic-6

For your Information:If time $t = 2\tau$ ($2RC$)

$$q = q_0(1 - e^{-\frac{2t}{\tau}})$$

Then $q = q_0(1 - e^{-2})$

$$q = 0.86q_0$$

This shows that the capacitor is charged about 86% after 2 time constant.

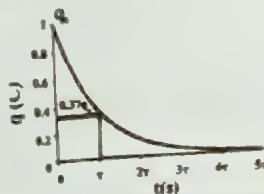
- The relation for discharging is given by

$$q = q_0 e^{-\frac{t}{\tau}}$$

If $t = 1\tau$ ($1RC$)

$$q = q_0 e^{-1} = 0.37q_0$$

This shows that discharging of capacitor also decreases exponentially and graphically represented by



It means a capacitor is discharged about 63% or there are 37% charges left on the capacitor

For your Information:If time $t = 2\tau$ ($2RC$)

$$q = q_0 e^{-2} = 0.14q_0$$

This shows that a capacitor is discharged about 86% after two time constants.

Time Constant

$$\tau = RC$$

Its unit is 's' i.e. second.

- Time constant is defined as the time required by a capacitor to charge up to 0.63 times the equilibrium charge on the capacitor.
- Charge reaches its equilibrium value **sooner** when time constant is smaller.
- Windshield wipers of car work by charging and discharging of capacitor.

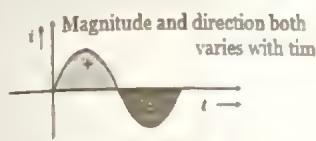
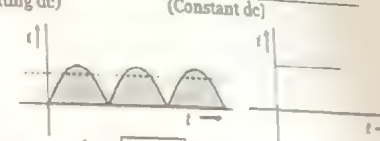


CRITICAL CONCEPT

After how many time constants the capacitor will almost discharged?

TOPIC-7 >> CURRENT ELECTRICITY**COURSE CONTENT**

- Ohm's Law
- Electrical resistance
- Specific resistance or resistivity
- Effect of temperature on resistance (Temperature coefficient of resistance or resistivity)
- Internal resistance of a supply
- Electric power (unit of electric power)
- Kilowatt-hours

Types of current:

Alternating current (ac)	Direct current (dc)
(i)  ac \rightarrow Rectifier \rightarrow dc	(i) (Pulsating dc)  (Constant dc) dc \rightarrow Inverter \rightarrow ac
(ii) Shows heating effect only	(ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is 	(iii) It's symbol is 

ELECTRIC CURRENT

- The rate of flow of charge in a circuit is defined as current.

$$\text{i.e. Current} = \frac{\text{Charge}}{\text{Time}} \quad \text{or} \quad I = \frac{Q}{t} \quad \text{or} \quad Q = It$$

Example: The current in a resistor is 8.0 mA. What charge flows through the resistor in 0.020 s?

- A) 0.16 mC B) 1.6 mC C) 4.0 mC D) 0.40 mC

Solution: B) $Q = It$

$$= (8.0 \text{ mA})(0.020 \text{ s})$$

$$= 0.16 \text{ mC}$$

- Electric current is equal to charge that flows in one second.
- Electric current is a scalar quantity.
- Unit of current is ampere in M.K.S. system and stat ampere in C.G.S. system.
and 1 ampere = 3×10^9 stat ampere
- 1 ampere = 6.25×10^{18} electrons / second.
- Direction of current is from higher to lower potential. The direction of current is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

- In metals electric conduction takes place due to flow of free electrons only. But in gases and electrolytes, electric conduction takes place due to flow of both positive and negative ions.

Conventional & Electronic Current

CONVENTIONAL CURRENT	ELECTRONIC CURRENT
Current due to positive charge is called conventional current.	Current due to electrons or negative charges is called electronic current.
It flows from higher to lower potential.	It flows from lower to higher potential.
It flows in direction along the direction of field.	It flows from lower to higher potential.

- Both conventional and electronic currents are flowing in a circuit, then total current will be given as; $I_{\text{total}} = I_{+ve} + I_{-ve}$
- Drift velocity (V_d)** It is the average velocity attained by free electrons on applying external electric field. In conductors, $v_d = 10^{-3} \text{ ms}^{-1}$
- Current density (J)** The current flowing per unit normal area of cross section is defined as current density. i.e. $J = \frac{I}{A} (\text{amp m}^{-2})$. Current density is a vector quantity. Its direction is from higher potential to lower potential or its direction is that of the flow of positive charge. $J = nev_d$ where v_d is the drift velocity of electrons.

For Your Information:

- An electric shock is a violent disturbance of the nervous system caused by an electrical discharge or current through the body.

CRITICAL THINKING?

- The diagram shows a model of an atom in which two electrons move round a nucleus in a circular orbit. The electrons complete one full orbit in $1.0 \times 10^{-15} \text{ s}$.



What is the current caused by the motion of the electrons in orbit?

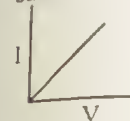
- A. $1.6 \times 10^{-34} \text{ A}$ B. $3.2 \times 10^{-34} \text{ A}$
C. $1.6 \times 10^{-4} \text{ A}$ D. $3.2 \times 10^{-4} \text{ A}$

OHM'S LAW

If the physical conditions of the conductor (temperature etc.) remain constant, then the applied potential difference is directly proportional to the current flowing in it i.e.,
 $I \propto V$ or $V = IR$

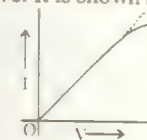
R is constant called the resistance of the conductor.

Ohmic substances The substances which obeys Ohm's law is called ohmic substances. Metals are ohmic substances. Graph for ohmic substances between V and I is a straight line.

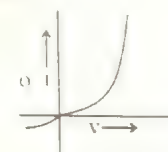


Ohm's law is valid only for those not obey Ohm's law is called **non-ohmic conductor**. Its $V-I$ graph is not a straight line.

Non-ohmic resistances Those resistances which do not obey Ohm's law are called non-ohmic resistances. Diode valve, triode valve, pentode, electrolytes, transistors, Torch bulb etc, are non-ohmic resistances. Graph between V and I for non-ohmic resistances is not a straight line but a curve. It is shown in figure.



For Filament



For Diode

- Slope of $I-V$ graph is equal to conductance. (If V along x-axis and I along y-axis)
- Slope of $I-V$ graph is equal to resistance. (If I along x-axis and V along y-axis)

For Your Information:

- Flow of current is directly proportional to the potential difference.
- Flow of heat is directly proportional to the temperature difference.

CRITICAL THINKING?

- Which one of the following material is non-ohmic:
 A. Gold B. Germanium
 C. Copper D. Silver

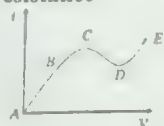
ELECTRICAL RESISTANCE

- It is the property of a conductor, by virtue of which it opposes the flow of current in it.
- $R = \frac{V}{I} = \frac{\text{Potential difference between the end of conductor}}{\text{Current flowing in the conductor}}$
- Unit of R is ohm in M.K.S. system and stat-ohm in C.G.S. system and $1 \text{ ohm} = \frac{1}{9 \times 10^{11}} \text{ stat ohm}$
- Unit ohm** The resistance of a wire, through which a current of 1 ampere flows on applying, a potential difference of 1 volt across its ends, is defined as one ohm.

Resistance depends upon

- The area of cross-section of the conductor ($R \propto \frac{1}{A}$)
 - The length of the conductor ($R \propto L$)
 - The nature of material of the conductor $\therefore R = \rho \frac{L}{A}$
- Where ρ is known as the specific resistance or resistivity.
- The temperature of the conductors.

Example: From the graph between current i & voltage V shown, identify the portion corresponding to negative resistance



- A) DE B) CD C) BC D) AB

Solution: $R = \frac{V}{i}$ in the graph CD has only negative slope. So in this portion R is negative

SPECIFIC RESISTANCE OR RESISTIVITY

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross-sectional area and unit length.

$$\rho = \frac{RA}{L}$$

so if $L = 1 \text{ m}$ and $A = 1 \text{ m}^2$ then $\rho = R$

- Unit of ρ is ohm \times meter
- Resistivity depends on

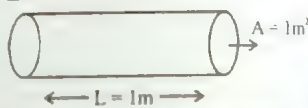
(a) nature of material (b) temperature.

Example: A wire $l = 8 \text{ m}$ long of uniform cross-sectional area $A = 8 \text{ mm}^2$ has a conductance of $G = 2.45 \Omega^{-1}$. The resistivity of material of the wire will be

- A) $2.1 \times 10^{-7} \Omega \text{ m}$ B) $3.1 \times 10^{-7} \Omega \text{ m}$ C) $4.1 \times 10^{-7} \Omega \text{ m}$ D) $5.1 \times 10^{-7} \Omega \text{ m}$

Solution: C) $\rho = \frac{RA}{l} = \frac{A}{Gl} = \frac{8 \times 10^{-6}}{2.45 \times 8} = 4.1 \times 10^{-7} \Omega \text{ m}$

- ρ does not depend on the size and shape of the material because it is the characteristic property of the conductor material.
- Order of specific resistance $\rho_{\text{insulator}} > \rho_{\text{semiconductor}} > \rho_{\text{conductor}}$
- The value of specific resistance (ρ) is minimum for Ag and maximum for silver, so $\rho_{\text{Ag}} > \rho_{\text{Cu}} > \rho_{\text{Al}}$. Hence Ag is best conductor and Cu is second best conductor of electricity.
- Constantan, Nichrome, Maganine, Ureka (alloys) and tungsten have high value of ρ , hence resistance of wires of these material is also high.



EFFECT OF TEMPERATURE ON RESISTANCE (TEMPERATURE CO-EFFICIENT OF RESISTANCE OR RESISTIVITY)

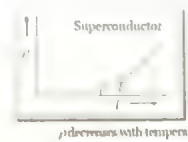
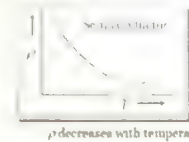
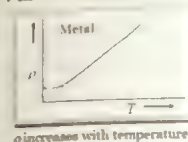
- Resistance of pure metal (e.g. copper, aluminum) increases with the increase of temperature. The change in resistance is fairly regular for normal range of temperatures.
- The resistance of electrolytes, insulators (e.g. glass, mica, rubber etc.) and semiconductors (e.g. germanium, silicon etc.) decreases with the increase in temperature.
- Resistance of constantan wire does not change with small change of temperature (Since temperature coefficient of resistance of constantan is very small). Hence standard resistance used in resistance boxes and post office box is made of constantan.
- The reciprocal of resistivity is called conductivity

$$\sigma = \frac{1}{\rho} \quad \text{and Unit of } \sigma \text{ is } \frac{1}{\text{ohm} \times \text{metre}} \text{ or } \frac{\text{mho}}{\text{metre}}$$

Temperature coefficient of resistance (α) It is the increase in resistance per unit original resistance at 0°C per unit rise (1°B) in temperature. $\alpha = \frac{R_1 - R_0}{R_0 \cdot t}$ Its unit is per $^\circ \text{C}$

For conductors α is positive i.e., on increasing temperature, resistance increase. For semiconductors (Si, Ge) and insulators (diamond quartz) α is negative, i.e., on increasing temperature resistance decreases.

Variation of resistivity with temperature



CRITICAL THINKING?

3. If the resistivity of the conductor is $2 \times 10^{-6} \Omega \text{ m}$ then its conductivity is
- A. $2 \times 10^6 \Omega^{-1} \text{ m}^{-1}$ B. $5 \times 10^6 \Omega^{-1} \text{ m}^{-1}$
C. $5 \times 10^{-5} \Omega^{-1} \text{ m}^{-1}$ D. $5 \times 10^5 \Omega^{-1} \text{ m}^{-1}$

INTERNAL RESISTANCE OF SUPPLY

emf of a source is defined as potential difference between its output terminals when either its internal resistance is zero or no current is being drawn from it.

- When charge carriers flow through a conductor, they lose their electrical K.E. In doing work against resistance, loss of energy is compensated by source of emf at same rate.
- Every source of emf has its own resistance called **internal resistance**.
- Smaller is the internal resistance of a battery, better it will be a source of emf.
- Terminal potential difference is a voltage between output terminals of a source of emf when current is drawn from it.**
- We can relate emf (\mathcal{E}), terminal potential (V_t) and internal resistance (r) by the following equation

$$V_t = \mathcal{E} - Ir \quad \text{OR} \quad \mathcal{E} = V_t + Ir$$

Example: A new flashlight cell of emf 1.5 volts gives a current of 15 amps, when connected directly to an ammeter of resistance 0.04 ohm. The internal resistance of cell is

- A) 0.04 ohm
B) 0.06 ohm
C) 10 ohm
D) 0.10 ohm

Solution: B)

$$i = \frac{E}{R + r} \Rightarrow 15 = \frac{1.5}{0.04 + r} \Rightarrow r = 0.06 \Omega$$

CRITICAL THINKING?

4. In the presence of internal resistance of the source, which one of the following relations between potential difference (V) and e.m.f (E) is correct
- A. $E = 0$
B. $E = V$
C. $E > V$
D. $E < V$

ELECTRICAL POWER

Joule Thomson's Effect & Power Dissipation

- When current flows through conductors, then a part of energy of current carriers is transferred to ions on their way, due to which their amplitudes of vibration increases which result in rise of temperature. The above phenomenon is known as *Joule Thomson's effect*. Hence electrical energy is wasted that is given as;

$$W = QV$$

$$P = \frac{W}{t} = \frac{QV}{t}$$

$$P = IV$$

$$P = I^2 R \Rightarrow P = \frac{V^2}{R}$$

Where P represents the power dissipation.

- Heat produced due to power dissipation is given as;

$$H = P \times t$$

$$= I^2 R t = V I t = V^2 t / R$$

- If 1J electrical energy is dissipated per second, then power dissipated will be 1 watt.
1 watt = 1J / 1s

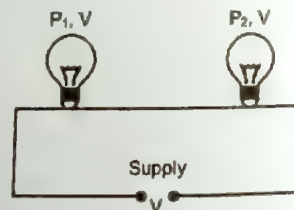
Combination of Bulbs:

Bulbs in Series:

- Total power consumed $\frac{1}{P_{\text{total}}} = \frac{1}{P_1} + \frac{1}{P_2} + \dots$

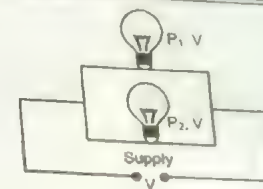
- If 'n' bulbs are identical, $P_{\text{total}} = \frac{P}{n}$

- P_{consumed} (Brightness) $\propto V \propto R \propto \frac{1}{P_{\text{rated}}}$ i.e. in series combination bulb of lesser wattage will give more bright light and p.d. appeared across it will be more.



Bulbs in Parallel:

- Total power consumed
- $P_{\text{total}} = P_1 + P_2 + P_3 + \dots + P_n$
- If 'n' identical bulbs are in parallel $P_{\text{total}} = nP$
- P_{consumed} (Brightness) $\propto P_R \propto i \propto 1/R$ i.e. in parallel combination, bulb of greater wattage will give more bright light and more current will pass through it.



Solved example: Two bulbs are working in parallel order. Bulb A is brighter than bulb B. If R_A and R_B are their resistance respectively then

- (A) $R_A > R_B$ (B) $R_A < R_B$ (C) $R_A = R_B$ (D) None of these

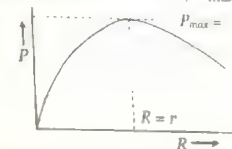
Solution: (B) In parallel $P_{\text{consumed}} \propto \text{Brightness} \propto 1/R$
 $P_A > P_B$ (given) $R_A < R_B$

- Kilowatt-hour** is a commercial unit of electrical energy. It is defined as "When a power of 1kW is maintained through a circuit for 1 hour, then energy dissipated is 1kWh."

$$1 \text{ kWh} = 1000 \text{ W} \times 3600 \text{ sec} = 3.6 \times 10^6 \text{ J}$$

$$J = 2.77 \times 10^{-7} \text{ kWh}$$

- Power delivered will be maximum when $R = r$, $P_{\text{max}} = E^2 / r$

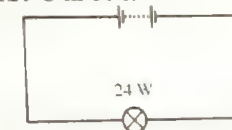


- Maximum power is delivered to a load R when the internal resistance of the source of emf is equal to the load resistance, also called maximum power transfer theorem. The value of the maximum output power is

$$(P_{\text{out}})_{\text{max}} = \frac{\epsilon^2}{4r} = \frac{\epsilon^2}{4R}$$

CRITICAL THINKING?

5. A battery is used to light a 24 W electric lamp. The battery provides a charge of 120 C in 60 s.



What is the potential difference across the bulb?

- A. 5 V
B. 12 V
C. 24 V
D. 120 V

TOPIC-8 >> ELECTROMAGNETISM

COURSE CONTENT

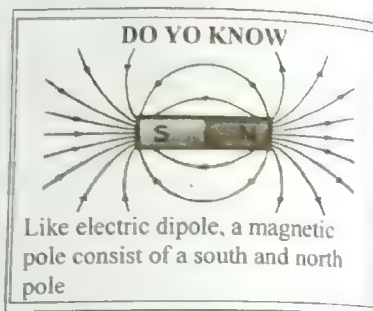
- Magnetic field
- Magnetic Flux and Magnetic Flux Density
- Force acting on a charged particle in a uniform magnetic field.
- Path followed by charge particle shot in the magnetic field in the direction perpendicular to the field.

MAGNETIC FIELD

- Study of magnetic properties is called magnetism.
- Study of magnetic properties associated with electricity and laws relating to them is called electromagnetism.
- Iron ore magnetite (Fe_3O_4) was discovered as early as 600 B.C from Magnesia, a region in Asia Minor, situated in modern Turkey.
- Magnetic field arises due to moving charges.

Characteristics of a Magnet

- It has two poles.
- North Pole of magnet coincides with the South Pole of earth's magnet & vice versa.
- Freely suspended magnet sets itself along N-S direction.
- An un-magnetized magnetic material can be magnetized as follows;
 - ✦ By electric method (passing strong D.C)
 - ✦ By single touch or double touch methods.
- Magnet can be demagnetized as follows;
 - ✦ By passing A.C
 - ✦ By heating strongly
 - ✦ By striking a magnet again and again with a surface like that of earth. e.g by hammering



Magnetic Effect of Current

When an electric current is passed through a conductor then a magnetic field is produced around the conductor. It is called the magnetic effect of current.

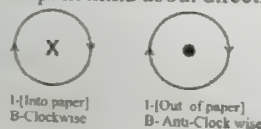
Magnetic field (B): The region of space around a magnet in which its magnetic effects are experienced, is defined as magnetic field.

- It is a vector quantity.
- Unit intensity of magnetic field (B) is weber m^{-2} or Newton/ampere x meter or tesla in M.K.S. system and Maxwell/ cm^2 or Gauss or Oersted in C.G.S. system.

Right Hand Rule

If the wire is grasped in fist of right hand with the thumb pointing in the direction of the conventional current, the fingers of the hand will circle the wire in the direction of the magnetic field

- Following figures should be kept in mind about direction of magnetic field.



CRITICAL THINKING?

1. Magnetic field lines form ____ loops from pole to pole.
 - A. Open
 - B. Closed
 - C. Branched
 - D. Either closed or branched

MAGNETIC FLUX AND MAGNETIC FLUX DENSITY

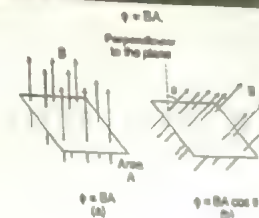
MAGNETIC FLUX

Magnetic flux is a measurement of the total magnetic field lines which passes through a given area. The scalar product of magnetic field strength (B) and the vector area (A) is

$$\phi = B \cdot A$$

If the magnetic field is not perpendicular to the plane, but makes an angle θ with the normal to the plane the flux linked with the plane will be

$$\phi = (B \cos \theta) A = BA \cos \theta$$



Maximum Flux:

If the angle between B and A is 0° the flux will be maximum

$$\phi_{\max} = BA$$

Minimum Flux:

If the angle between B and A is 90° then magnetic flux will be zero

$$\phi_{\min} = 0$$

Unit of Magnetic Flux:

The unit of magnetic flux ϕ is weber.

Magnetic flux density:

The magnetic induction B is the flux per unit area of a surface perpendicular to B is also called as flux density.

- **Formula:** $B = \frac{\phi}{A}$

Unit:

- weber metre $^{-2}$
- Newton/amp-meter.
- Unit of B is also called tesla. In C.G.S. system unit of magnetic flux density is gauss
 $1 \text{ tesla} = 10^4 \text{ gauss}$

FORCE ACTING ON A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Magnetic Force on a Moving Charge

When a charged particle of charge q is moving with velocity v in a magnetic field B at an angle θ , then force acting on the particle:

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \quad \text{or} \quad F = qvB \sin \theta$$

Where θ is angle between \vec{v} and \vec{B} .

- Direction of force F is perpendicular to both v and B. this direction can be found out by "Fleming's left hand rule".

For electron $F = -e(\vec{v} \times \vec{B})$ and for proton $F = +e(\vec{v} \times \vec{B})$

CRITICAL THINKING?

2. A 2MeV proton is moving perpendicular to a uniform magnetic field of 2.5T. The force on the proton is:
 A. $2.5 \times 10^{-10} \text{N}$ B. $7.6 \times 10^{-11} \text{N}$
 C. $2.5 \times 10^{-11} \text{N}$ D. $7.6 \times 10^{-12} \text{N}$
3. A proton is moving along z-axis in a magnetic field. The magnetic field is along x-axis. The proton will experience a force along
 A. x-axis B. y-axis
 C. z-axis D. -ve z-axis

PATH FOLLOWED BY CHARGE PARTICLE IN MAGNETIC FIELD

- If $v = 0$, then $F = 0$
- If $\theta = 0^\circ$, then $F = qvB \sin 0 = 0$
 So the particle will continue to move in straight line with the same speed.
- If $\theta = 180^\circ$, then $F = qvB \sin 180$ or $F = 0$
 Here also the particle will move in same direction in straight line with same speed.
- If $\theta = 90^\circ$, then $F = qvB \sin 90$ or $F_{\max} = qvB$
 In this case the charged particle will move in circular path and the plane of the circle is perpendicular to B.

$$qvB = \frac{mv^2}{r}$$

$$\text{or } r = \frac{mv}{qB}$$

r is called radius of circular path or cyclotron radius.

$$\text{Angular velocity or angular frequency of the particle: } \omega = \frac{v}{r} = \frac{qB}{m}$$

$$\text{So, } \omega = \frac{qB}{m} \quad \text{But } \omega = 2\pi f$$

$$\text{or } f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

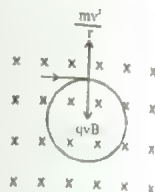
Also time period of particle $T = \frac{1}{f} = \frac{2\pi m}{qB}$ and kinetic energy of particle

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m \left(\frac{rqB}{m} \right)^2$$

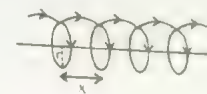
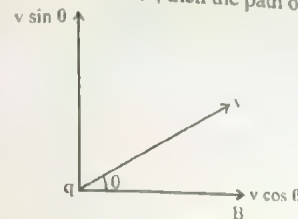
$$\text{so } K.E = \frac{r^2 q^2 B^2}{2m}$$

CRITICAL CONCEPT

A force exerts on a moving charged particle in a magnetic field, but in what direction it moves that the force does not exert on it?



If θ lies between 0° and 90° , then the path of the particle is helical or helix



Here radius of helix $r = \frac{mv \sin \theta}{qB}$ and pitch of helix

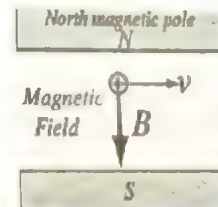
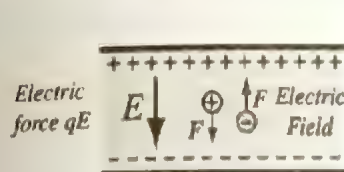
$$x = v \cos \theta \times T = v \cos \theta \times \frac{2\pi m}{qB} \quad T = \frac{2\pi m}{qB}$$

Force on Charge Particle in An Electric and Magnetic Field

$$\text{As, } \vec{F}_e = q\vec{E}, \text{ and } \vec{F}_m = q(\vec{v} \times \vec{B})$$

$$\text{or } \vec{F} = \vec{F}_e + \vec{F}_m$$

$$\text{or } \vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$



Magnetic force of magnitude $qvB \sin \theta$ perpendicular to both v and B , away from viewer.

Where \vec{F} is called Lorentz force. If charge is of mass 'm' in uniform electric field, then

$$a = \frac{\vec{F}}{m}$$

- Electric force does work
- Magnetic force does no work

Example: A particle of charge $-16 \times 10^{-18} \text{C}$ moving with velocity 10 m/s along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

- A) 10^3Wb m^{-2} B) 10^4Wb m^{-2}
 C) 10^{16}Wb m^{-2} D) 10^{-3}Wb m^{-2}

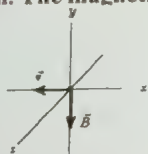
$$\text{Solution: A) } \vec{F} = \vec{F}_e + \vec{F}_m, \vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0 \Rightarrow B = \frac{E}{v} = 10^4 \text{Wb m}^{-2}$$

Difference between Electric and Magnetic Forces

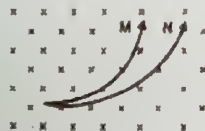
Sr.No.	Electric Force	Magnetic Force
1	Electric force on a charged particle is always collinear with the field.	Magnetic force on a charge particle is always perpendicular to the field.
2	Electric force (qE) is independent of the state of rest or the motion of the charged particle.	Magnetic force is velocity dependent i.e. acts only when the charged particle is in motion.
3	Electric force does work in displacing the charged particle.	Magnetic force does no work when the charged particle is displaced.
4	Electric force, $\vec{F} = q\vec{E}$	Magnetic force, $\vec{F} = q(\vec{v} \times \vec{B})$ $= qvB \sin \theta$ When $\theta = 0$ Force = 0 When $\theta = 90^\circ$ $F = qvB$ (max)
5	Electric force may or may not be non-central.	Magnetic force is always non-central.

CRITICAL THINKING?

4. An electron moves in the negative x direction, through a uniform magnetic field in the negative y direction. The magnetic force on the electron is



- A. In the negative z direction
B. In the positive z direction
C. In the negative y direction
D. In the positive y direction
5. Two charged particles M and N are projected with same velocity in a uniform magnetic field. Then M and N are:



- A. A proton and α -particle respectively
B. A deuteron and an electron respectively
C. An electron and a proton respectively
D. A protium and a proton respectively

e/m FOR AN ELECTRON

e/m of a charged particle can be given as; $\frac{e}{m} = \frac{v}{Br}$

Where R is determined by Thomson's apparatus while velocity is determined by following two methods:

Potential difference method: $v = \sqrt{2V_0 \frac{e}{m}}$

so that $\frac{e}{m} = \frac{2V_0}{B^2 r^2}$

Velocity selector method: $v = \frac{E}{B}$

so that, $\frac{e}{m} = \frac{E}{B^2 r}$

The value of $\frac{e}{m}$ is equal to 1.7588×10^{11} C/kg

Example: Four particles independently move at the same speed in a direction perpendicular to the same magnetic field. Which particle is deflected the most?

- A) a copper ion
B) a helium nucleus
C) an electron
D) a proton

Solution: The force exerted on a particle by the magnetic field is given by Bqv . As this force provides the necessary centripetal force to deflect the particle.

$$Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq}$$

For a large deflection, r is small. Among the four particles, the electron has the lowest $\frac{m}{q}$ ratio.

CRITICAL THINKING?

6. An electron enters a region where the electric field E is perpendicular to the magnetic field B . It will suffer no deflection if

- A. $E = Bev$
B. $B = eE/v$
C. $E = Bv$
D. $E = Bev/2$



COURSE CONTENT

- Electromagnetic induction
- Faraday's Law
- Lenz's Law and conservation of energy
- Alternating Current Generator
- Transformers

ELECTROMAGNETIC INDUCTION

- "When the magnetic flux linking a conductor changes, an e.m.f is induced in the conductor, this phenomenon is known as electromagnetic induction". Induction is the change in flux linking the conductor (or coil)
- The product of number of turns (N) of the coil and the magnetic flux (Φ) linking it is called flux linkages i.e. Flux linkages = $N\Phi$

DO YOU KNOW



Electronic card swapping system based on the electromagnetic induction theory

FARADAY'S LAW

Faraday's Laws of Electromagnetic Induction

Faraday has given the following two laws regarding electromagnetic induction

- Whenever there is a change in the magnetic flux linked with a circuit, an induced emf is produced in the circuit. If the circuit is closed, an induced current flows through it. The current flows only so long as the magnetic flux is changing.
- The induced emf is equal to the negative rate of change of magnetic flux. Thus, if $\Delta\Phi$ is the change in flux in time interval Δt , then the induced emf in the circuit is

$$\epsilon = -\frac{\Delta\Phi}{\Delta t}, \text{ N number of turns, } \epsilon = -N \frac{\Delta\Phi}{\Delta t}$$

- The negative sign shows the induced emf opposes the change in magnetic flux.
- If rate of change of magnetic flux be in weber/sec, the induced emf will be in volt.
- If the coil contains N turns $\epsilon = -N \frac{\Delta\Phi}{\Delta t} = \frac{\Delta(N\Phi)}{\Delta t}$

Example: A flat circuit coil of 120 turns, each of area 0.70 m^2 , is placed with its axis parallel to a uniform magnetic field. The flux density of the field is changed steadily from 80 mT to 20 mT over a period of 4.0 s . Induced emf will be
A) 0 B) 130 mV C) 170 mV D) 210 mV

Solution: B) $\epsilon = -N \frac{\Delta B}{\Delta t}$

$$= 120 \times 0.070 \times \frac{(80 - 20) \times 10^{-3}}{4.0}$$

$$= 0.126 \text{ V}$$

$$= 130 \text{ mV}$$

Induced Current

Induced emf in a coil is given by $\epsilon = -N \frac{\Delta\Phi}{\Delta t}$. If the resistance of the coil is R

of its circuit be R , then the induced current in the circuit is $I = \frac{\epsilon}{R} = \frac{N \Delta\Phi}{R \Delta t}$

Induced Charge

The charge flowing through the circuit in time Δt is

$$q = I \times \Delta t = \frac{N \Delta\Phi}{R \Delta t} \times \Delta t = \frac{N}{R} \Delta\Phi$$

CRITICAL THINKING?

- Emf (ϵ_1) is induced in circular coil of 100 turns in which magnetic flux changes from 20 Wb to 40 Wb in 1 s . While emf (ϵ_2) is induced in another coil of same number of turns in which magnetic flux changes from 30 Wb to 150 Wb in 1 minute which of following is correct.

- A. $\epsilon_1 > \epsilon_2$
C. $\epsilon_1 < \epsilon_2$

- B. $\epsilon_1 > \epsilon_2$
D. $\epsilon_1 = \epsilon_2$

LENZ'S LAW AND CONSERVATION OF ENERGY

According to Lenz's rule the induced current produced in a closed circuit is in such a direction that it opposes the cause due to which it has been produced.

- In general, the direction of induced emf of current is determined by Fleming's right hand rule. In case of motion of straight conductor in a magnetic field, it can be also be determined by "Fleming's right hand rule".
- When the north pole of a magnet moves towards a stationary loop, an induced current flows in the anticlockwise sense as seen from the side on which the magnet is moving. The anticlockwise sense corresponds to the generation of north pole (N) which opposes the motion of the approaching N-Pole of the magnet.



When the N pole of the magnet is moved away from the coil, the current I flows in the clockwise sense which corresponds to the generation of south-pole (S). The induced field opposes the motion of receding N-pole.



Lenz's law and the induced current opposes the motion of magnet. Hence some mechanical work has to be done on the magnet to move it. This work is converted into electrical energy. The work performed on the coil is the induced EMF.

A closed circuit may be connected to the coil of magnet that will allow the generated EMF to drive a current. This current will work against a force that is being exerted by the magnet to move it. The induced EMF is the electromotive force.

CRITICAL THINKING?

1. If current in conductor increases, then according to Lenz's law self-induced voltage will
 - a) increase the increasing current
 - b) tend to decrease the increasing current
 - c) produce current opposite to increasing current
 - d) tend to oppose it
2. In the diagram shown if a bar magnet is moved along the common axis of two single turn coils A and B in the direction of arrow
 - a) Current is induced only in A & not in B
 - b) Induced currents in A & B are in the same direction
 - c) Induced currents in A & B are in opposite directions
 - d) Current is induced only in B and not in A

ALTERNATING CURRENT GENERATOR

AC generator is a device which converts mechanical energy into electrical energy in the presence of magnetic field.

- The principle of generator is the coil placed between a magnet. Part of coil is in magnetic field.
- Main parts of AC generator
 - Armature coil (rotor) which rotates with constant speed.
 - Permanent magnets, W and NW iron cylinders.
 - Slip rings or commutator.
 - Carbon brush (electrical supply).
- EMF is induced by the side of coil intersecting the magnetic field.
- Induced EMF is number of lines of force.

$$\epsilon = N \frac{d\Phi}{dt} (\sin \theta)$$

also

$$\epsilon = N \omega AB \sin \omega t$$

$$\epsilon_{\max} = N \omega AB$$

when $\theta = 90^\circ$

then

$$\epsilon = \epsilon_{\max} \sin \omega t$$

If $\omega = 2\pi f$, then

$$\epsilon = \epsilon_{\max} \sin 2\pi ft$$

In terms of potential difference, $V = V_0 \sin 2\pi ft$

In terms of current, $I = I_0 \sin 2\pi ft$

where f is the frequency with which the direction of current is changing.



- In Pakistan $f = 50$ Hz. It means 50 times in a second direction is changing.

For your information:

The electric generator can generate AC or DC and some special

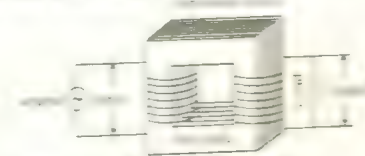
CRITICAL THINKING?

1. In ac generator, ac current reverses its direction
 - a) 10 times per second
 - b) 20 times per second
 - c) 50 times per second
 - d) 40 times per second

TRANSFORMER

Transformer

It is a device which rises or lowers the voltage in A.C. circuits through mutual induction and consists of two coils wound on the same core. The coil which is connected to the source (i.e., to which input is applied) is called primary, while the other which is connected to the load (i.e., from which output is taken) is called secondary. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an electromotive force in the secondary. As magnetic lines of force are closed lines, the flux primary is the same as the flux in the secondary.



And as the number of turns in each coil is constant

$$\frac{\Phi_1}{N_1} = \frac{\Phi_2}{N_2} \quad \text{or} \quad \frac{E_1}{N_1} = \frac{E_2}{N_2} \quad \left[\text{as} \quad \epsilon = \frac{d\Phi}{dt} \right]$$

And as the induced EMF is the same in both coils, $E_1 = E_2$ and $V_1 = V_2$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

Now there are two possibilities

- $N_s > N_p$ the transformer is said to be "step up" and it increases voltage and reduces current
- $N_s < N_p$ the transformer is said to be "step down" and it reduces voltage and increases current.
- Transformer works on A.C. only and never on D.C.
- It can increase or decrease either voltage or current but not both simultaneously (as power = constant)

Example: The secondary coil of an ideal transformer delivers an r.m.s current of 2.5 A to a load resistor of resistance 8.0 Ω . The r.m.s current in the primary is 10 A.

What is the r.m.s. potential difference across the primary coil?

- A) 3.5 V B) 5.0 V C) 57 V D) 80 V

Solution: B) r.m.s potential difference across the secondary coil

$$= (2.5)(8)$$

$$= 20 \text{ V}$$

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} \Rightarrow V_p = \frac{I_s}{I_p} \times V_s = \frac{2.5}{10} \times 20 = 5.0 \text{ V}$$

Losses in Actual Transformer

The losses in transformer occur in core and winding.

Types of Losses in Transformer

- **Copper Losses in Winding**
Due to resistance of the windings of primary and secondary coils, some electrical energy is always lost in the form of heat energy.
- **Flux Losses**
The coupling of the primary and secondary is never perfect and whole of the magnetic flux produced in the primary coil does not link the secondary coil. This results in some energy loss.
- **Iron Losses in Core**
Iron losses are of two types: Eddy current loss and hysteresis loss.
 - (i) Eddy Current Loss**
Due to the periodically varying nature of A.C. supplied in primary, the flux associated with core changes and, therefore, eddy currents are induced in it.
Eddy currents induced in the core are undesirable as they heat the core and result in energy loss. To minimize the eddy current losses, core is laminated.
 - (ii) Hysteresis Loss**
The alternating current flowing through the coils magnetizes and demagnetizes the iron core again and again. Therefore, during each cycle of magnetization, some energy is lost due to hysteresis. To minimize this loss we choose material of core of smaller hysteresis loss generally soft iron.

Efficiency of Transformer

Ideal transformer, efficiency is 100 % or 1. but in actual transformer output power is always less than input power, so efficiency also always less than 100%. In general efficiency of transformer is very high (and is of the order of 90%).

Efficiency is given by $\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$. In terms of secondary and primary

voltages and currents, $\eta = \frac{V_s I_s}{V_p I_p}$. Also since, $\text{Input} = \text{Output} + \text{Losses}$, so

$$\eta = \frac{\text{Output power}}{\text{Input power} + \text{Losses}}$$

CRITICAL THINKING?

- An iron -cored transformer with 50% efficiency has ratio of the secondary turns to the primary turns is 1: 20. A 240 V ac supply is connected to primary coil and a 6 Ω resistor is connected to secondary coil what is the current in the primary coil?
A. 0.2A B. 0.1A
C. 1A D. 1.4A
- A loss free transformer has 500 turns on its primary winding and 2500 in secondary. The meters of the secondary indicate 200 volts at 8 amperes under these conditions. The voltage and current in the primary is
A. 100 V, 16 A B. 160 V, 10 A
C. 40 V, 40 A D. 80 V, 20 A

COURSE CONTENT

RECTIFICATION

Introduction

- Rectification is the process of converting AC into DC.
- The process of converting AC into DC is called rectification.

Half-Wave Rectification (HWR)

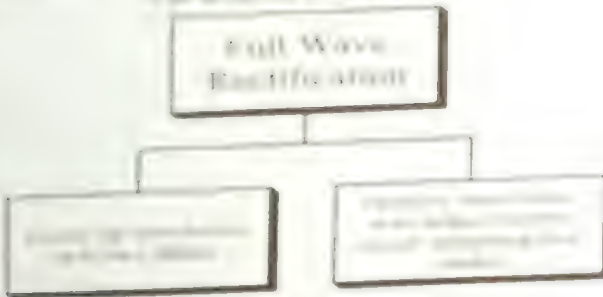
- The diode is used.
- The diode is connected in series with the load.
- The diode is forward biased during the positive half cycle of the AC.
- The diode is reverse biased during the negative half cycle of the AC.

Draw Blocks

- The diode is connected in series with the load.
- The diode is forward biased during the positive half cycle of the AC.
- The diode is reverse biased during the negative half cycle of the AC.

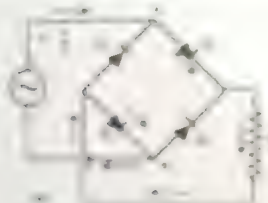
Full-Wave Rectification (FWR)

Full-Wave Rectification



Draw Circuit

- The diode is connected in series with the load.
- The diode is forward biased during the positive half cycle of the AC.
- The diode is reverse biased during the negative half cycle of the AC.



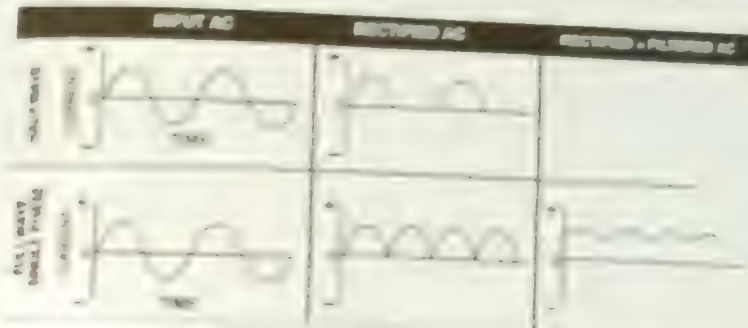
Summary

- 1. In a full-wave rectifier, the diode is forward biased during the positive half cycle of the AC and reverse biased during the negative half cycle of the AC.
- 2. The output of a full-wave rectifier is a pulsating DC voltage.

Example: If a full-wave rectifier circuit is operating from 50 Hz mains, the fundamental frequency in the ripple will be

- A) 50 Hz
- B) 100 Hz
- C) 25 Hz
- D) 75 Hz

Solution: C) In full-wave rectifier, the fundamental frequency in ripple is twice that of input frequency.



Advantages

- It is a simple and efficient method of rectification.
- It requires less number of diodes.
- Lesser ripples are there in the signal compared to Half-Wave Rectification.

Example: The electrical current used to get smooth DC output from a rectifier circuit is called

- A) Filter
- B) Rectifier
- C) Load
- D) Transformer

Solution: A)

Filter is the component which is used to remove the ripples from the rectified output.

Q.10.10

- 1. The diode shown in the circuit is a silicon diode. The potential difference between the points A and B will be



TOPIC-11 » DAWN OF MODERN PHYSICS

COURSE CONTENT

Photon and particle model of light in terms of photons with particular energies

PHOTON

The Photon

- Einstein presented the idea of light energy consisting of packets of electromagnetic energy.
- Max plank explained the emission and absorption by the atoms from a black surface in the form of indivisible packets called **quanta**.
- Max plank put discontinuous (**granular**) nature of light.
- The beam of light with wavelength λ consists of stream of photons traveling at speed c and carrier energy hf .
- Emission or absorption of energy is applied to **any oscillating system**.
- Einstein defined light in terms of photon which is called photon (particle) theory of light.
- The particle nature of light has been observed in the Compton effect and practically has been proved in Davisson and Germer experiment.

CRITICAL CONCEPT

The rest mass of photon is zero and its momentum also zero?

Salient features of photon:

- A photon behaves as a particle whose rest mass is zero and it travels with speed of light $3 \times 10^8 \text{ ms}^{-1}$. In other words, a photon exists as long as it is moving. It ceases to exist when it comes to rest.
- Photons are electrically neutral and are not deflected in the presence of electric and magnetic fields.
- The energy of a photon is given as:

$$E = hf = \frac{hc}{\lambda} \therefore f\lambda = c$$

This shows that the energy of photon depends upon frequency (or wavelength).

- Momentum of photon is given as:

$$\text{As } E = hf = mc^2$$

$$\text{So } mc = \frac{hf}{c}$$

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

- Rest mass of a photon can be calculated by using Einstein's mass variation equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow m_0 = m \sqrt{1 - \frac{v^2}{c^2}}$$

Since photon is moving with speed of light ' c ' so $v = c$
This shows that rest mass of a photon is zero.

Example: An air station is broadcasting the waves of wavelength 300 m. If the radiating power of the transmitter is 10 kW, then the number of photons radiated per second is

A) 1.5×10^{29}

B) 1.5×10^{31}

C) 1.5×10^{33}

D) 1.5×10^{35}

Solution: B) If n be the number of photons/s, then $E = \frac{nhc}{\lambda}$

$$\frac{nhc}{\lambda} = 10 \times 10^3 = n = \frac{10 \times 10^3 \times 300}{6.6 \times 10^{-34} \times 3 \times 10^8} = \frac{3 \times 10^6}{6.6 \times 3 \times 10^{-26}} = \frac{1}{6.6} \times 10^{32} = 1.5 \times 10^{31}$$

CRITICAL THINKING?

1. What happens to the energy of a photon when it strikes matter?
- It turns into heat
 - It turns into electrical energy
 - The matter emits electrons
 - All of the above

CRITICAL CONCEPT

If an electron and a proton having same wavelength. Which particle has higher speed?

Example: A tiny particle of mass 10^{-13} kg moving with a velocity of 10 cm s^{-1} is associated with a wave of wavelength

A) $6.62 \times 10^{-18} \text{ cm}$

C) $6.62 \times 10^{-2} \text{ cm}$

B) $6.62 \times 10^{-12} \text{ cm}$

D) $6.62 \times 10^{-34} \text{ cm}$

Solution: A) $\lambda = \frac{h}{mv}$

$$= \frac{6.62 \times 10^{-34}}{10^{-13} \times 10 \times 10^{-2}} \times 100 \text{ cm} = 6.62 \times 10^{-18} \text{ cm}$$

CRITICAL THINKING?

2. Protons & Alpha particles have same wavelength, what is same for both of them?
- energy
 - Time period
 - Frequency
 - momentum
3. A body of mass 200 g moves at the speed of 5 m/hr. So wavelength related to it is of the order ($h = 6.26 \times 10^{-34} \text{ Js}$)
- 10^{-10} m
 - 10^{-30} m
 - 10^{-20} m
 - 10^{-40} m

Topic-11

PHOTOELECTRIC EFFECT AND PHOTON THEORY OF LIGHT

The emission of electrons from metallic surface, when light of specific short wavelength is incident on it is called "Photoelectric effect".

Hallwach applied some potential difference across two Zn plates in a quartz vacuum tube and studied the flow of current. When ultraviolet light is incident on cathode current flows in the circuit which vanishes when no light falls. When light falls on anode the current in the circuit is negligible.

Effect of Intensity of Incident Light on Photoelectric Current: When the intensity of light frequency more than the threshold frequency is increased the number of photoelectrons increases, i.e., the photoelectric current also increases.

Photoelectric current, $i \propto I$ where $I = \text{Intensity of light}$

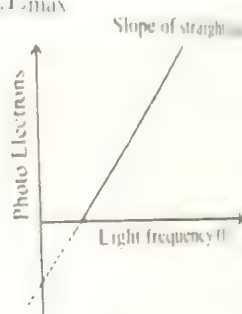
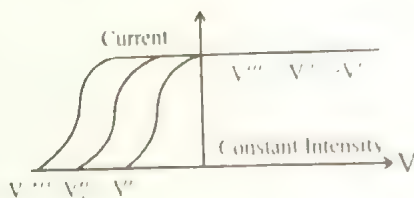
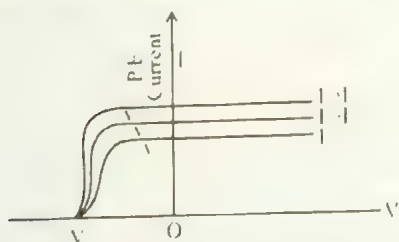
Effect of Potential on Photoelectric Current

- On increasing the potential current first increases and at a fixed potential reaches a maximum value known as saturation current.
- At a fixed negative potential the value of photoelectric current is zero. This negative potential is called stopping potential.
- The stopping potential is proportional to the maximum kinetic energy of photoelectrons.
- The stopping potential depends on the frequency of the incident light.
- The stopping potential does not depend on the intensity of the incident light.

Effect of Frequency of Incident Light on Photoelectric Current

A simple linear relation exists between stopping potential (maximum energy of emitted electron) and frequency of incident photon.

$$hf = eV + \phi_0 \quad \text{or} \quad hf = K.E. + \phi_0 \quad \text{or} \quad \phi_0 = hf - K.E._{\text{max}}$$



Laws of Photoelectric Effect

- The rate of emission of photoelectrons from a metallic surface is proportional to the intensity of incident light.
- If the frequency of incident light is less than a specific minimum (whatever the intensity of light) electrons will not be ejected from the surface. This minimum (threshold) frequency is different for different metals. The photo energy corresponding to threshold frequency is known as work function of metal.

$$\phi_0 = hf_0 = \frac{hc}{\lambda_0}$$

- The maximum K.E. of emitted photoelectrons is proportional (linearly related) to the frequency of incident light but does not depend on the intensity of incident light.
- The time interval of incidence of light on the metallic surface and electron emission is negligible (less than 10^{-8} s). i.e., the process of electron ejection is instantaneous.

Parameters of Photoelectric Effect

- Work function (ϕ_0):** The minimum energy required to eject an electron from the metallic surface is known as its work function.

$$\phi_0 = hf_0$$

(a) It depends upon:

- The impurities present on the surface of the metal
- The nature of metal

(b) Its unit are eV, joule and erg

(c) It is a property of material and not of emitted electron

Photo sensitive material: Those elements that emit photons when high frequency light is incident on the material are called photo sensitive material.

Saturated photocurrent: The maximum value of photocurrent is called saturated current.

Due to stopping potential the work done by electrons is equal to the maximum kinetic energy of electrons:

$$eV_s = \frac{1}{2}mv_{\max}^2$$

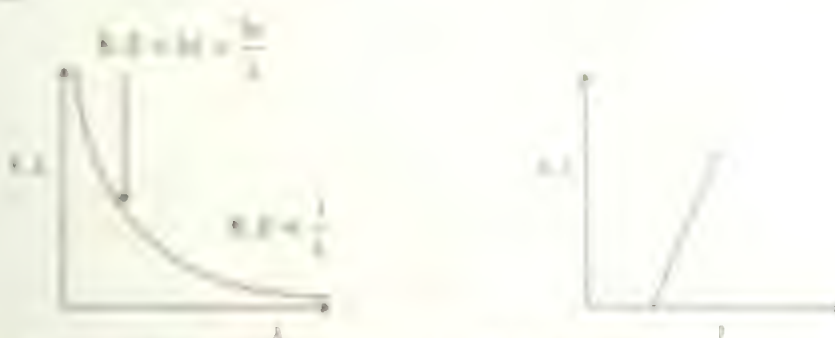
Einstein's Explanation of Photoelectric Effect: Light has dual (wave and particle) nature. In interaction of light with matter light acts as particle. According to quantum theory the exchange or propagation of light is in the form of small energy packets called photons.

$$E = hf = \frac{hc}{\lambda}$$

Einstein's Photoelectric Equation:

$$K.E._{\max} = hf - \phi = hf - hf_0 = h(f - f_0) \quad \frac{1}{2}mv_{\max}^2 = hf - \phi$$

Important Graphs:



Failure of Classical Theory to Explain Photoelectric Effect:

The wave theory of light completely failed to explain the experimentally established facts about photoelectric effect.

- The fact that maximum kinetic energy of photoelectrons does not depend on intensity of incident radiation.
- The existence of a threshold frequency or wavelength.

A weak beam of radiations having frequency more than threshold frequency can eject a photo electron while an intense beam of frequency lesser than threshold cannot eject a photo electron.

Compton Effect:

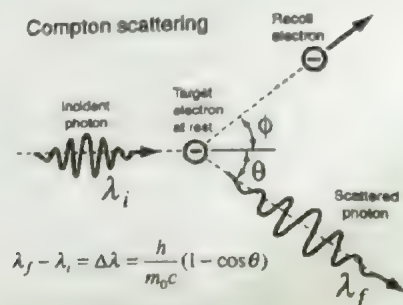
When a photon hits with an electron, it scatters with frequency less than that of incident photon. It is known as Compton effect.

- Usually X-ray photons are used because of high energy (≥ 17.5 keV).
- Change (increase) in wavelength is called Compton shift.

$$\Delta\lambda = \lambda' - \lambda$$

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos\theta)$$

- $\Delta\lambda = \frac{h}{m_0c} = 2.43 \times 10^{-12} \text{ m}$ is called **Compton wavelength**.
- $\Delta\lambda = 0$ when $\theta = 0^\circ$.
- $\Delta\lambda = \frac{h}{m_0c}$ when $\theta = 90^\circ$.
- $\Delta\lambda = \frac{2h}{m_0c}$ when $\theta = 180^\circ$.

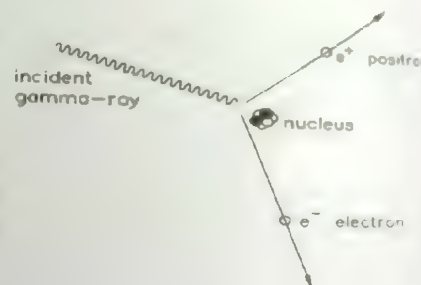


- Photoelectric effect and Compton effect are strong evidences that e.m waves behave as **particle (photon)**.
- Compton effect proves **photon** theory of light.

Pair Production:

Decomposition of photon into electron, positron pair is called pair production

- Pair production can take place only if photon energy is greater than **1.02 MeV**.
- Energy equation for pair production is given as;
 $hf = 2m_0c^2 + K.E_{e^-} + K.E_{e^+}$
- Rest mass energy of electron or positron is m_0c^2 ($= 0.51 \text{ MeV}$).
- Condition for pair production is that $hf > 2m_0c^2$
- Pair production cannot take place in **vacuum**
- The interaction usually takes place in the electric field in the vicinity of a heavy nucleus so that there is a particle to take up recoil energy and momentum is conserved.
- Pair production is **materialization of energy**

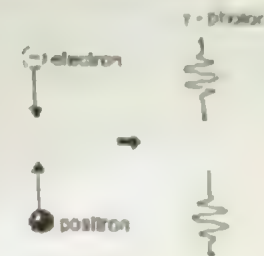


Annihilation of Matter:

Reverse process of pair production is called **annihilation of matter**.

- It involves conversion of mass into energy.
- Two photons are produced by the annihilation of electron and positron
- Two photons produced move in opposite direction to obey the law of conservation of **momentum**.

- Each photon has energy of **0.51 MeV** equivalent to rest mass energy of electron



Antimatter:

- P.A.M Dirac theoretically predicted antimatter in 1928.
- Anderson discovered positron during study of spectrum of cosmic rays in 1932
- Every antiparticle has same mass, same spin but opposite charge to its respective particle

PARTICLE	ANTIPARTICLE
Electron	Positron
Proton	Antiproton
Neutron	Antineutron
Neutrino	Antineutrino
Barion	Antibaryon

de-Broglie's hypothesis (wave particle duality):

All the moving particles behave as waves called **matter waves** or **de-Broglie waves**. The wavelength associated with moving particles is given by

$$\lambda = \frac{h}{mv}$$

mv = momentum of particles

$$\lambda = \frac{h}{p}$$

$$\lambda \propto \frac{1}{m}$$

$$\lambda \propto \frac{1}{v}$$

An object of large mass and ordinary speed has such a small wavelength that such as interference and diffraction are negligible

Davisson and Germer Experiment:

- Germer and Davisson using low energy electron beam showed that electrons behave in exactly the same manner as X-rays or any other waves.

- The electron beam of energy V eV is made incident on a nickel crystal. The beam is scattered. The wavelength associated with the moving electrons is given as

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m \sqrt{2meV}}$$

where V is accelerating potential

$$\lambda = \frac{h}{m \sqrt{2meV}}$$

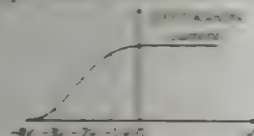
$$\lambda = \frac{1.24}{\sqrt{V}}$$

$$\lambda = \frac{1.24}{\sqrt{154}}$$

- The beam of electrons scattered from crystal surface was obtained for a glancing angle of 90° . According to Bragg's equation $2d \sin \theta = n\lambda$ the glancing angle is 45° .
 $d = 0.91 \times 10^{-10} \text{ m}$
 $\lambda = 0.91 \times 10^{-10} \text{ m}$
- Princess Maria Theresia de Broglie received the 1929 noble prize in physics Clinton Anderson and George Paget Thomson shared the Nobel Prize in 1937 for their experimental confirmation of the wave nature of particles.
 Bragg's equation is a practical application of wave particles duality.

CRITICAL THINKING?

4. The value of stopping potential in the following diagram



5. If the de-Broglie wavelengths for a proton and for a α -particle are equal, then the ratio of their velocities will be
 A. 1:2 B. 1:4
 C. 2:1 D. 4:1
6. In Compton scattering, the change in wave length is max. if
 A. angle of scattering is 90° B. angle of scattering is 60°
 C. angle of scattering is 180° D. angle of scattering is zero

COURSE CONTENT

Atomic Spectra Line Spectrum

ATOMIC SPECTRA / LINE SPECTRUM

Spectrum

Spectrum means set of frequencies absorbed or emitted by a substance

Types

Emission Spectrum

- (a) Set of frequencies emitted by atoms of a substance.

Absorption spectrum

- (b) Set of frequencies absorbed by atoms of a substance.

Absorption spectrum is caused by up transition of atomic electrons.

Each element has characteristic emission spectra in its vapour state.

Emission spectra fall into following three categories

- (a) Continuous spectra

It is emitted from condensed matter (solid or liquid). A black body spectrum is continuous spectra

- (b) Line spectra

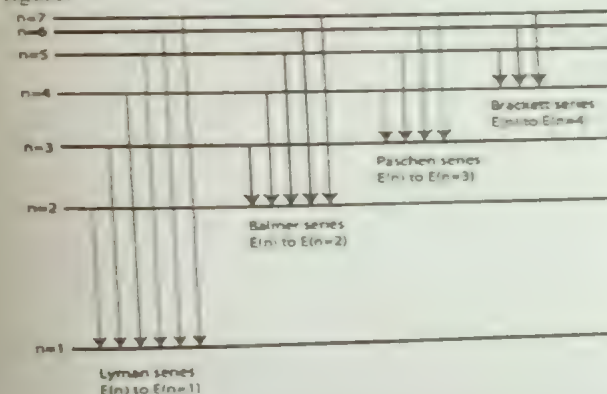
It is emitted by a gas or vapour state of element.

- (c) Band spectra

Molecular spectra is band spectra

Spectrum of Hydrogen

Whenever an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two levels is emitted as a radiation of particular wavelength. It is called a spectral line. Depending upon the order of their wavelengths, these spectral lines can be grouped into a number of spectral Series as shown in figure.



SERIES	RELATIONS	REGION	LONGEST WAVELENGTH	SHORTEST WAVELENGTH
LYMAN SERIES	$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$ $n = 2, 3, 4, \dots, \infty$	U.V	Put $n = 2$ $\lambda = \left(\frac{4}{3R} \right)$	Put $n = \infty$ $\lambda = \frac{1}{R}$
BALMER SERIES	$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$ $n = 3, 4, 5, \dots, \infty$	Visible	Put $n = 3$ $\lambda = \frac{36}{5R}$	Put $n = \infty$ $\lambda = \frac{4}{R}$
PASCHEN SERIES	$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right)$ $n = 4, 5, 6, \dots, \infty$	Infrared	Put $n = 4$ $\lambda = \frac{144}{7R}$	Put $n = \infty$ $\lambda = \frac{9}{R}$
BRACKET SERIES	$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right)$ $n = 5, 6, 7, \dots, \infty$	Infrared	Put $n = 5$ $\lambda = \frac{400}{9R}$	Put $n = \infty$ $\lambda = \frac{16}{R}$
PFUND SERIES	$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right)$ $n = 6, 7, 8, \dots, \infty$	Infrared	Put $n = 6$ $\lambda = \frac{900}{11R}$	Put $n = \infty$ $\lambda = \frac{25}{R}$

Example: If the wavelength of photon emitted due to transition of electron from third orbit to first orbit in a hydrogen atom is λ , then the wavelength of photon emitted due to transition of electron from fourth orbit to second orbit will be

- A) $\frac{128}{27}\lambda$ B) $\frac{25}{9}\lambda$ C) $\frac{36}{7}\lambda$ D) $\frac{125}{11}\lambda$

Solution: A) $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = R \left(1 - \frac{1}{9} \right) = \frac{8R}{9}$ or $\lambda = \frac{9}{8R}$

Again $\frac{1}{\lambda'} = R \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$

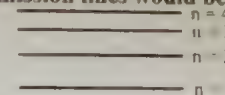
$\frac{1}{\lambda'} = R \left(\frac{1}{4} - \frac{1}{16} \right) = R \times \frac{3}{16} \Rightarrow \lambda' = \frac{16}{3}R$

$\frac{\lambda'}{\lambda} = \frac{16}{3R} \times \frac{8R}{9}$

$\lambda' = \frac{128}{27}\lambda$

CRITICAL THINKING?

1. Four lowest energy levels of H atom are shown in the figure. The number of possible emission lines would be



- A. 3 B. 4
C. 5 D. 6

2. Figure shows the energy levels P, Q, R, S and G of an atom where G is the ground state. A red line in the emission spectrum of the atom can be obtained by an energy level change from Q to S. A blue line can be obtained by following energy level change



- A. R to S B. Q to R
C. R to G D. P to Q

CRITICAL CONCEPT

What is the energy of the photon emitted when a hydrogen atom makes a transition from $n = 2$ to $n = 1$ state?

COURSE CONTENT

- Simple Model for the atom to include protons, neutrons and electrons
- Spontaneous and random nuclear decay/ the Law of Radioactive Decay
- Half Life and rate of decay
- Biological effects of Radiation
- Biological and Medical Uses of Radiation

ATOMIC NUCLEUS

- The nucleus is very small part which exists at the center of the atom.
- Nucleus was discovered by Rutherford through his α -scattering experiments.
- The whole positive charge and almost the whole mass of an atom resides inside the nucleus.
- The charge on the nucleus is $(+Ze)$. It is due to protons present in the nucleus.
- The radius of the nucleus is the order of 10^{-15} to 10^{-14} m. (fermi)
- If the nucleus is presumed to be spherical its radius $r = r_0 A^{\frac{1}{3}}$ where $r_0 = 1.2 \times 10^{-15}$ m and A is atomic number.
- The constituents of nucleus are neutrons and protons. In an atom electron equal in number to protons, revolve round the nucleus.
- In lighter nuclei the proton number equals the neutron number ($N = Z$) e.g. ${}^4_2\text{He}$ etc.
- In heavier nuclei the number of neutrons is greater than the number of protons ($N > Z$) e.g. ${}^{238}_{92}\text{U}$, ${}^{294}_{90}\text{Th}$ etc.
- The neutrons and protons present inside the nucleus taken together are known as nucleons.

Isotopes

The atoms of an element having same atomic number but different atomic mass number are called isotopes, e.g.

Hydrogen ${}_1\text{H}^1$, ${}_1\text{H}^2$, ${}_1\text{H}^3$

and

Oxygen: ${}_8\text{O}^{16}$, ${}_8\text{O}^{17}$, ${}_8\text{O}^{18}$

- The mass numbers (i.e. number of nucleons) of all isotopes of an element are different. Hence their physical properties are not the same.
- Among isotopes of the same element, some may be stable and some radioactive. This is due to difference in their nuclear structure. For example ${}_6\text{C}^{12}$ is stable while ${}_6\text{C}^{14}$ is radioactive similarly ${}_{11}\text{N}^{23}$ is stable while ${}_{11}\text{Na}^{24}$ is radioactive.

Isobars

- The nuclei having same number of nucleons (A) but different number of protons (Z) are called isobars. They also have different number of neutrons. For example
 (a) ${}_1\text{H}^3$ and ${}_2\text{He}^3$ (b) ${}_6\text{C}^{14}$ and ${}_7\text{N}^{14}$ (c) ${}_8\text{O}^{17}$ and ${}_9\text{F}^{17}$
- They differ in chemical properties.
- Isobars differ in physical properties also.

Nuclei of isobars belong to different elements.

The daughter nucleus remaining after emission of β -particles is an isobar of the parent nucleus.

Isotones

The nuclei having equal number of neutrons are called isotones. For them both Z and A are different but $(A - Z)$ is same. For example

(a) ${}_3\text{Li}^7$ and ${}_4\text{Be}^8$, $A - Z = 4$

(c) ${}_{11}\text{Na}^{23}$ and ${}_{12}\text{Mg}^{24}$, $A - Z = 12$

(b) ${}_1\text{H}^1$ and ${}_2\text{He}^3$, $A - Z = 2$

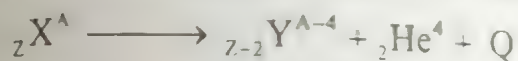
RADIOACTIVITY:

Henry Becquerel discovered **radioactivity**.

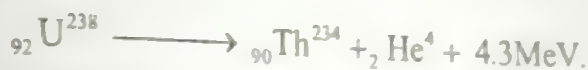
Nuclei having $Z > 82$ are unstable and they emit α , β and γ rays: the phenomenon is called **radioactivity**.

Some radioactive elements are Po ($Z=84$) Rd ($Z=88$) and U ($Z=92$) etc.

α -decay Charge no. (Z) decreases by 2 and mass no. (A) by 4



e.g.



β -particle β -particle is electron or positron coming from nucleus. So it may be classified as.



Only charge no. (Z) **increases by 1**.

e.g.



Note Weak interaction only appears in β -decay.

γ -decay γ -rays are massless photons; their emission will cause no change either in A or Z of the parent nuclide. γ -decay is due to de-excitation of nucleus.



where $*$ sign show excitation of atom

Example: A nucleus of the nuclide ${}_{94}^{241}\text{Pu}$ decays by emission of a β^- particle followed by the emission of an α^- particle. Which of the nuclides shown is formed?

A) ${}_{93}^{239}\text{Np}$

C) ${}_{93}^{237}\text{Np}$

B) ${}_{91}^{239}\text{Pa}$

D) ${}_{92}^{237}\text{U}$

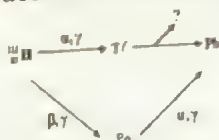
Solution: C) ${}_{94}^{241}\text{Pu} \rightarrow {}_y^x\text{A} + {}_{-1}^0\beta + {}_2^4\text{He}$

$$241 = x + 0 + 4 \Rightarrow x = 237$$

$$94 = y + (-1) + 2 \Rightarrow y = 93$$

Type of radiation	Alpha particle	Beta particle	Gamma ray
Symbol	α or ${}^4_2\text{He}$	β or β^-	γ (can look different, depends on the font)
Mass (atomic mass units)	4	1/2000	0
Charge	+2	-1	0
Speed	slow	fast	very fast (speed of light)
Ionizing ability	high	medium	0
Penetrating power	low	medium	high
Stopped by	paper	aluminium	lead

1. Part of the actinium radioactive series can be represented as follows:



A. α -particle

B. β -particle

C. One α -particle and two β -particles

D. Two β -particles

SPONTANEOUS AND RANDOM NUCLEAR DECAY

- Radioactive elements disintegrate and emit α , β and γ radiations. This process is called transmutation by spontaneous disintegration.
- In the natural spontaneous disintegration of a radioactive material not all the atoms disintegrate at the same time.
- The process of disintegration takes place randomly, when a nucleus disintegrates, nobody knows

HALF-LIFE AND RATE OF DECAY

- Radioactive decay is a random process so we get idea of **half-life**.
 - The half-life $T_{1/2}$ of a radioactive element is that period in which half of the atoms decay.
 - Half-life ($T_{1/2}$) depends on element and is not affected by any physical change or chemical change.
-
- Half-life may be classified as follows
 - (a) Extremely short half-life $T_{1/2}$ in micro or nano seconds
 - (b) Moderate half-life $T_{1/2}$ in minutes.
 - (c) Long half-life $T_{1/2}$ in years.

Half-life can be given as

$$\lambda T_{1/2} = \ln 2 = 0.693$$

Where λ is called decay constant depends upon nature of material. Decay constant of any element is equal to the fraction of the decaying atoms per unit time. The unit of the decay constant is s^{-1} .

The decay curve shows that radioactive element decays exponentially.

Half-life is used to identify an atom.

Laws of Radioactivity

- $\Delta N / \Delta t \propto N_0$
- $\Delta N / \Delta t = -\lambda N_0$
- $N_t = e^{-\lambda t} N_0$
- $T_{1/2} \propto 1/\lambda$
- $\lambda = \frac{-\Delta N / \Delta t}{N}$
- Mean life = $T^* = 1/\lambda$
- $T_{1/2} = T^* (0.693)$

Example: The half-life of a certain radioactive element is such that 7/8 of a given quantity decays in 12 days. What fraction remains undecayed after 24 days?

- A) 0 B) $\frac{1}{128}$ C) $\frac{1}{64}$ D) $\frac{1}{32}$

Solution: C) The radioactive element has 1/8 of a given quantity remains after 12 days. After 24 days, or in additional 12 days, the fraction remains undecayed is

$$\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) \text{ or } \frac{1}{64}$$

Half Life of some Radioactive Elements

Radioactive Nuclides	Half life
Iodine-131 (I)	8 days
Krypton-86 (Kr)	3.16 minutes
Sodium -24 (Na)	15 hours
Cobalt-60 (Co)	5.27 years
Radium-228 (Ra)	1600 years
Uranium-235 (U)	703 million years
Radium-226 (Ra)	1.6×10^3 years
Uranium-238 (U)	4.5×10^9 years
Plutonium-239	2.4×10^4 years
Radon-220	52 seconds
Strontium-90 (Sr)	28 years
Protactinium (Pa)	6.66 hours
Radon gas	3.8 days
Uranium-239	23.5 minutes

For Your Information:

- Any quantity that decreases by half over equal time intervals is said to decay exponentially.
- Any quantity that increases by twice over equal time interval is said to grow exponentially.

2. The percentage of the original quantity of a radioactive material left after five half-lives is approximately

A. 1 %
B. 5 %
C. 3 %
D. 20 %

3. When an animal dies each gram of carbon in its body emits about 16β particles each minute. Each gram of carbon from same animal remains is found to emit 4β particles per minute. How old is the animal (Half-life of radioactive carbon is 6000 years)?

A. 3000 years
B. 6000 years
C. 12000 years
D. 18000 years

- Radiations can damage living tissues. The degree of damage and kind of damage depends on type, energy and dose of radiation. Incident radiations ionize the body cell, and then change the biochemistry of the cell. A damaged cell may die or begins to work in wrong way. Sometimes radiation changes the chemistry of cells in such a way that they begin to reproduce rapidly leading to a condition called cancer.
- Damage to ozone layer is done through following sources: -
 - (i) Chemical industry
 - (ii) Nuclear tests
 - (iii) CFC (chlorofluorocarbons)
 - (iv) Aerosol sprays and plastic foam industry
- Some radiation in the environment is added by human activities like diagnostic x-ray, nuclear facilities, hospitals, research and industrial establishments, colour television, luminous watches and tobacco leaves.

These effect (alter) the chemistry of genes and cause mutation e.g. **cancer, different syndromes**
etc

- C-14 is one of useful tracer that can help in the understanding of **photosynthesis**.
- I-131 and Sr-90 are used to check **cracks in pipes**.
- **Radiation therapy** is a process of destruction of cancerous cells deep into body e.g. Co-60 and I-131 are used to treat cancers.

- Tumors are treated by γ -rays.
- Radio processes in space give information about **structure of stars**.
- When ethylene is radiated with γ -rays, it is polymerized into polyethylene, which is used to **produce soft and flexible products**.
- **Sterilization** is a process of killing of germs with β -particles or γ -rays.
- Radiation treatment can **preserve food** and other eatables.
- γ -gauges are used for preparing or measuring the thickness of high density and thicker materials such as **steel, Al & rubber**. In such gauges, Co-60 is used as γ -rays source.
- β -gauges are used for preparing or measuring the thickness of thin sheets of low density materials e.g. paper. In **such gauges, Sr - 90 is used as a β -rays source**
- **Radiography** is employed to check cracks of cavities in metal casting, faults in welding & heavy machinery.
- Ratio of C-14 to C-12 found in dead matter is a measure of time span **since death**.
- **β particle is used for superficial skin therapy.**
- **α particle is used for deep skin therapy.**
- Activation analysis (γ -ray energy measurement) is applied to determine concentration of elements in a given sample and to estimate corrosion and wear of machinery.
- Technetium - 99 has given rise to positron emission tomography.
- The ratio of ${}_{62}\text{Pb}^{206}$ — ${}_{92}\text{U}^{238}$ is used to determine age of rocks.

Topic -1 Force and Motion

Displacement (d):

$$d = v \times t$$

$$d = \frac{v}{2} \times t$$

Units:

metre (m)

Dimensions:

[L]

Velocity:

$$v = \frac{\Delta d}{\Delta t}$$

$$v = \frac{\text{Total displacement}}{\text{Total time}} = \frac{d}{t}$$

$$v = \frac{\Delta d}{\Delta t}$$

$$v_1 = \frac{2v_2}{1 - v_2/c^2} \quad \text{if } (v_1 < c)$$

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \quad \text{if } (v_1 < c)$$

$$v_1 = \frac{v_2 + v_3}{1 + \frac{v_2 v_3}{c^2}} \quad \text{if } (v_1 < c)$$

Relative velocity:

$$v_{AB} = v_A - v_B = 2v \cos \theta$$

$$v_{AB} = v_A - v_B \quad (\theta = 90^\circ)$$

$$v_{AB} = v_A - v_B \quad (\theta = 0^\circ)$$

$$v_{AB} = v_A - v_B \quad (\theta = 180^\circ)$$

Units:

Dimensions:

Acceleration:

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v}{t}$$

$$a = \frac{v^2}{r}$$

Units:

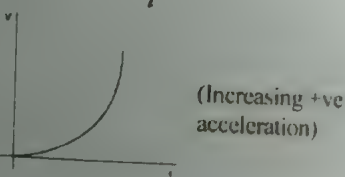
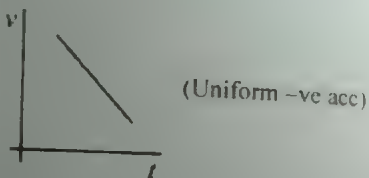
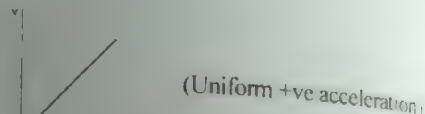
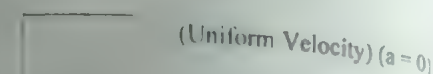
$$m/s^2$$

Dimensions:

$$[L T^{-2}]$$

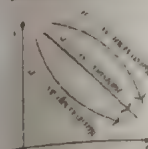
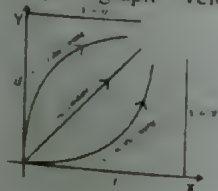
Graphical representation of acceleration with velocity-time graph:

- Slope of graph = Acceleration
- Area under graph = Distance

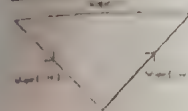


Displacement-time graph:

Slope of graph = velocity



General slope tree:



Newton's laws of motion:

$$F = ma$$

$$\frac{F_1}{F_2} = \frac{m_1}{m_2} \quad (a = a)$$

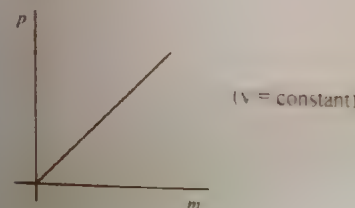
$$\frac{F_1}{F_2} = \frac{a_1}{a_2} \quad (m = m)$$

$$\frac{m}{m_2} = \frac{a}{a_2} \quad (F = F)$$

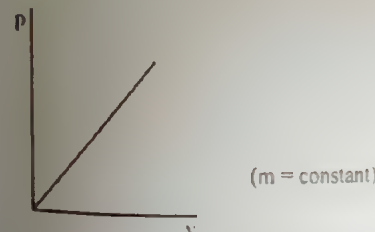
Linear Momentum:

$$p = mv$$

$$p \propto m$$



$$p \propto v$$



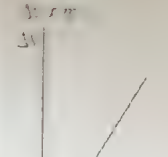
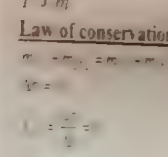
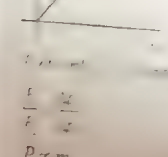
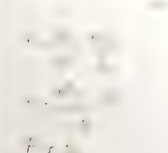
Important Graphs & Formulae

When body dropped:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$



Momentum - time graph:



Slope = $\frac{p}{t} = \frac{mv}{t}$ Force = F

Projectile Motion:

$v_x = v \cos \theta$

$v_y = v \sin \theta$

$v_y = v \sin \theta - gt$

$v_y = \sqrt{v_y^2 + v_x^2}$

$v = \sqrt{v_x^2 \cos^2 \theta + v_x^2 \sin^2 \theta + g^2 t^2 - 2v \sin \theta (gt)}$

$v = \sqrt{v_x^2 + g^2 t^2 - 2v \sin \theta gt}$

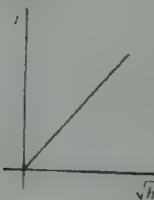
Vertical distance:

$y = S = \frac{1}{2} gt^2 = 5t^2$

$t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{h}{5}}$

$t \propto \sqrt{h}$



Horizontal distance:

$x = v_x \times t = v_x \sqrt{\frac{2h}{g}}$

$x \propto \sqrt{h}$

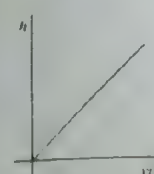
$x \propto \sqrt{h}$

Maximum Height:

$h = \frac{v_y^2 \sin^2 \theta}{2g} = \frac{v_y^2}{2g}$

$h \propto \sin^2 \theta$

$h \propto v^2$



$h_{\max} = \frac{v_y^2}{2g}$

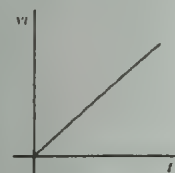
$h = h_{\max} \sin^2 \theta$

Time of flight:

$t = \frac{2v_y \sin \theta}{g} = \frac{2v_y}{g}$

$t \propto \sin \theta$

$t \propto v_y$



- Time taken by projectile from place of projection to maximum height.

$t_1 = \frac{t}{2} = \frac{v_y \sin \theta}{g} = \frac{v_y}{g}$

- Time taken by projectile from maximum height to place of landing.

$t_2 = \frac{t}{2} = \frac{v_y \sin \theta}{g}$

$t_{\max} = \frac{2v_y}{g}$

$t = t_{\max} \sin \theta$

Horizontal Range:

$R = (t)(v_x)$

$R = t \times v_x \cos \theta$

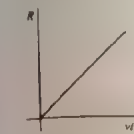
$R = \frac{v_y^2 \sin^2 2\theta}{g}$

$R = \frac{2(v_y \sin \theta)(v_x \cos \theta)}{g}$

$R = \frac{2(v_{ix})(v_{iy})}{g}$

$R_{\max} (\theta = 45^\circ)$

$R \propto v_y^2$



$R_{\max} = \frac{v_y^2}{g}$

$R = R_{\max} \sin^2 \theta$

$R \propto \sin^2 \theta$

For two angles, if

$\theta_1 + \theta_2 = 90^\circ$ then $R_1 = R_2$

(If " v_i " for both projectiles is same)

K.E at maximum height:

$K.E_h = KE \cos^2 \theta$

P.E at maximum height:

$P.E = KE \sin^2 \theta$

Momentum at maximum height:

$p_h = p \cos \theta$

$p_h = mv \cos \theta = mv_x$

Relations between R, H and t:

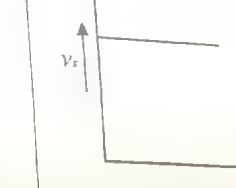
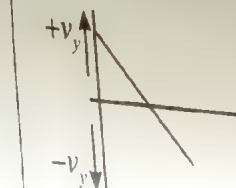
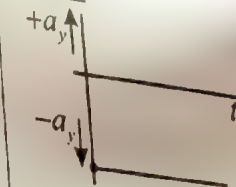
$R \tan \theta = 4H$

$R_{\max} = 4H (\theta = 45^\circ)$

$H \propto \left(\frac{g}{8}\right)^2$

$H \propto t^2$

Graph:



Topic - 2 Work and Energy

Work:

$$W = \vec{F} \cdot \vec{d}$$

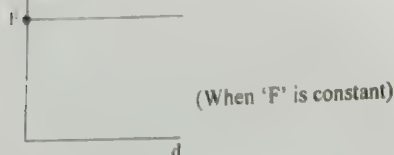
$$W = Fd \cos \theta$$

$$W_{\max} = +Fd \quad (\theta = 0^\circ)$$

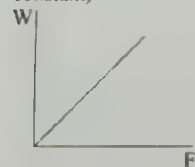
$$W_{\min} = -Fd \quad (\theta = 180^\circ)$$

$$W_{\min} = 0 \quad (\theta = 90^\circ)$$

Graph:



$W \propto F$ (When 'd' & 'θ' are constant)



$W \propto d$
(When 'F' is constant)

Area = Fd = Work

Units:

- joule (S.I unit)
- kilowatt hour (kW h)
- calories (cal)
- erg (c.g.s)
- electron volt (eV)
- 1kW h = 3.6 MJ
- 1cal = 4.18 J
- 1erg = 10^{-7} J
- 1eV = 1.6×10^{-19} J

Dimension:

$$[W] = [ML^2 T^{-2}]$$

Work done by Variables Force:

$$W = F_1 \Delta d_1 \cos \theta_1 + F_2 \Delta d_2 \cos \theta_2 + \dots + F_n \Delta d_n \cos \theta_n$$

$$W = \sum_{i=1}^n F_i \Delta d_i \cos \theta_i$$

Power:

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t} = \frac{\vec{F} \cdot \vec{\Delta d}}{\Delta t} = \frac{mgh}{t} = \frac{P.E}{t} = \frac{K.E}{t} = \frac{1/2 mv^2}{t} = \frac{mv^2}{2t}$$

$$P_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t}$$

$$P_{\text{inst}} = \vec{F} \cdot \vec{v}$$

$$= Fv \cos \theta$$

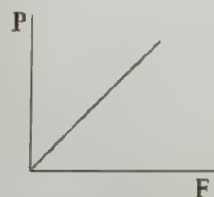
$$P_{\text{inst}} = Fv \quad (\theta = 0^\circ)$$

$$P_{\text{inst}} = -Fv \quad (\theta = 180^\circ)$$

$$P_{\text{inst}} = \vec{\tau} \cdot \vec{\omega} = \tau \omega \cos \theta$$

When 'v' & 'θ' are constant

$$P \propto F$$



Energy:

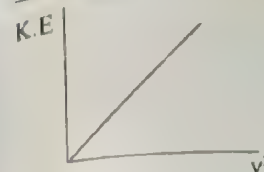
$$G.P.E = mgh$$

$$\text{Elastic P.E} = \frac{1}{2} kx^2$$

$$\text{Electric P.E} = q\Delta V$$

$$K.E = \frac{1}{2} m(v, \vec{v}) = \frac{1}{2} mv^2$$

$$K.E \propto v^2 \quad (\text{when } m = \text{constant})$$

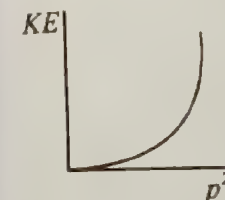


K.E and momentum:

$$K.E = \frac{p^2}{2m}$$

$$K.E = \frac{1}{2} pv$$

$$K.E \propto p^2$$



$$\frac{K.E_1}{K.E_2} = \left(\frac{m_1}{m_2} \right) \left(\frac{p_1}{p_2} \right)^2$$

$$\frac{K.E_1}{K.E_2} = \left(\frac{p_1}{p_2} \right)^2 \quad (m = \text{constant})$$

$$p = \sqrt{2m K.E}$$

$$\frac{p_1}{p_2} = \sqrt{\frac{K.E_1}{K.E_2}} \quad (m = \text{constant})$$

$$p \propto \sqrt{K.E}$$

Interconversion of K.E and P.E:

In the absence of Air;

Loss in P.E = Gain in K.E

$$mgh = \frac{1}{2} mv^2$$

$$\sqrt{2gh} = v$$

In the presence of air;

Loss in P.E = Gain in K.E + Work done against friction

$$mgh = \frac{1}{2} mv^2 + fh$$

$$mgh - fh = \frac{1}{2} mv^2$$

Topic - 3

Rotational and Circular Motion

Angular displacement (Revolution,

Degree, Radian):

$$s = r\theta$$

$$\theta = \frac{s}{r}$$

$$1^\circ = 0.0174 \text{ rad}$$

$$180^\circ = \pi \text{ rad}$$

$$1 \text{ rad} = 57.3^\circ$$

$$1 \text{ rev} = 2\pi \text{ rad}$$

Angular Velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$v = \omega \times r = \omega r \sin 90^\circ (\hat{n})$$

$$v = r\omega$$

$$r = \frac{v}{\omega}$$

$$\omega = \frac{v}{r}$$

Units:

- degree/s
- radian/s (S.I unit)
- rev/min

$$1 \text{ rev/min} = \frac{2\pi}{60} \text{ rad/s} = \frac{\pi}{30} \text{ rad/s}$$

Dimensions:

$$[T^{-1}]$$

Angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\alpha_{\text{avg}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} = \alpha r \sin 90^\circ (\hat{n})$$

$$a = r\alpha$$

$$a = \frac{v^2}{r}$$

$$\omega = \frac{v}{r}$$

Units:

- degree/s²

$$\bullet \text{ rad/s}^2$$

$$\bullet \text{ rev/min}^2$$

$$\bullet 1 \text{ rev/min}^2 = \frac{2\pi}{(60)^2} \text{ rad/s}^2 = \frac{\pi}{1800} \text{ rad/s}^2$$

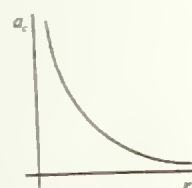
Dimensions:

$$[T^{-2}]$$

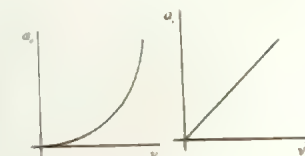
Centripetal acceleration a_c :

$$a_c = \frac{v^2}{r} \Rightarrow \vec{a}_c = -\left(\frac{v^2}{r^2}\right)\vec{r} = -\omega^2\vec{r}$$

$$a_c \propto \frac{1}{r} \quad (v = \text{Constant})$$

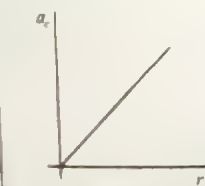


$$a_c \propto v^2 \quad (r = \text{Constant})$$

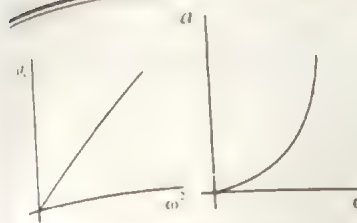


$$a_c = r\omega^2$$

$$a_c \propto r \quad (\omega = \text{Constant})$$



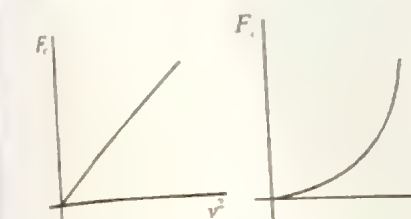
$$a_c \propto \omega^2 \quad (r = \text{Constant})$$



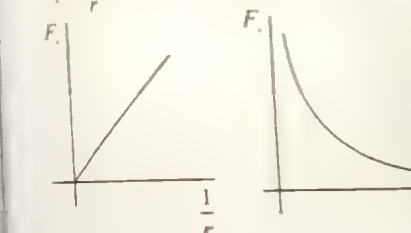
Centripetal force F_c :

$$F = m \frac{v^2}{r} \rightarrow \vec{F}_c = -\left(\frac{mv^2}{r}\right)\vec{r} = -\left(\frac{mv^2}{r^2}\right)\vec{r}$$

$$F_c \propto v^2 \quad (r = \text{Constant})$$

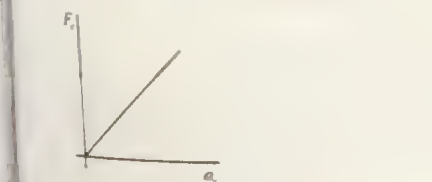


$$F_c \propto \frac{1}{r} \quad (v = \text{Constant})$$



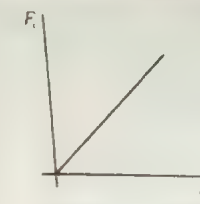
$$F = ma_c$$

$$F_c \propto a_c \quad (m = \text{Constant})$$

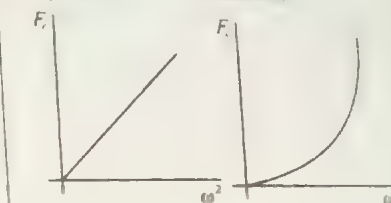


$$F_c = mr\omega^2 \quad \text{or} \quad \vec{F}_c = -(m\omega^2)\vec{r}$$

$$F_c \propto r \quad (\omega = \text{Constant})$$



$$F_c \propto \omega^2 \quad (r = \text{Constant})$$



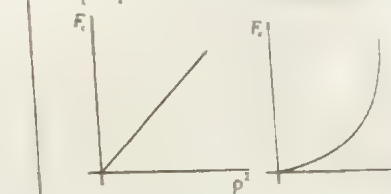
$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{2mv^2}{2r}$$

$$F_c = \frac{2K.E}{r}$$

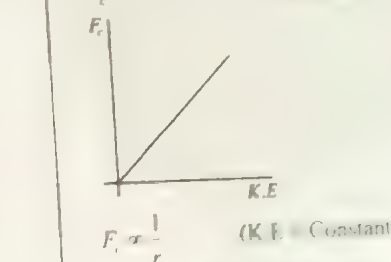
$$F_c = \frac{p^2}{mr}$$

$$F_c \propto p^2 \quad (mr = \text{Constant})$$

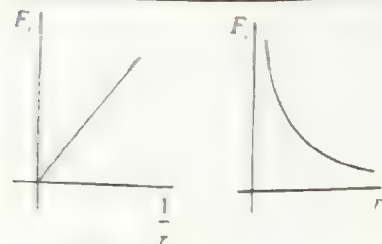


$$F_c = \frac{2K.E}{r}$$

$$F_c \propto K.E \quad (r = \text{Constant})$$



$$F_c \propto \frac{1}{r} \quad (K.E = \text{Constant})$$



$$F_c = \frac{mv^2}{r} = \frac{m \cdot 4\pi^2 r}{T^2}$$

$$F_c = 4\pi^2 m f^2 r$$

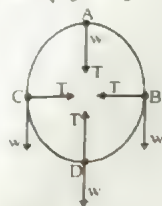
$$a_c = \frac{4\pi^2 r}{T^2}$$

$$a_c = 4\pi^2 f^2 r$$

Vertical circle:
Non-uniform circular motion:

$$T = F_c - w \cos \theta$$

$$v = \sqrt{3gr - 2gr \cos \theta}$$



At A:

- $\theta = 0$
- $v = \sqrt{gr}$
- $T_{\text{min}} = F_c - w$

At B:

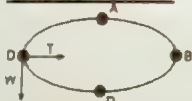
- $\theta = 90$
- $v = \sqrt{3gr}$
- $F_c = T$

At D:

- $\theta = 180$
- $v = \sqrt{5gr}$
- $F_c = T - w$

- Note:**
- θ is angle between T and w

Horizontal circle:



At A, B, C, D

- $\theta = 90^\circ$
- $F_c = T$
- $v = \sqrt{3gr}$

Topic - 4 Waves

In-phase:

$$\text{Path difference} = x = n\lambda$$

$$\text{Minimum path difference} = x = 0\lambda$$

$$n = 0, 1, 2, 3, \dots$$

$$\text{Phase difference} = \phi = \frac{2\pi x}{\lambda}$$

$$\phi = 0, 2\pi, 4\pi, 6\pi, \dots$$

$$\phi = 2n\pi$$

$$\phi_{\text{min}} = 0$$

Out of phase:

$$\text{Path difference} = x = \left(n + \frac{1}{2}\right)\lambda = (2n+1)\frac{\lambda}{2}$$

$$\text{Minimum path difference} = x = \frac{\lambda}{2}$$

$$\text{Phase difference} = \phi = \frac{2\pi x}{\lambda}$$

$$\phi = \pi, 3\pi, 5\pi, \dots$$

$$\phi_{\text{min}} = \pi$$

$$\phi \propto x$$



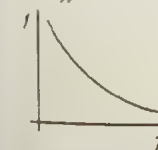
Wave speed v:

$$v = f\lambda = \frac{\lambda}{T}$$

For same medium,

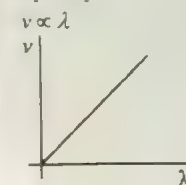
$$\frac{f_1}{f_2} = \frac{\lambda_2}{\lambda_1} \quad (v = \text{constant})$$

$$f \propto \frac{1}{\lambda}$$

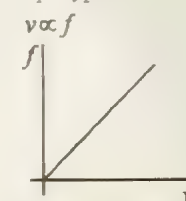


For different medium,

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad (f = \text{constant})$$



$$\frac{v_1}{v_2} = \frac{f_1}{f_2} \quad (\lambda = \text{constant})$$



Principle of superposition:

Interference

$$Y_o = y_1 + y_2 + y_3 + \dots + y_n$$

In phase	Out phase
$A = a_1 + a_2$	$A = a_1 - a_2$
$I_{\text{max}} = (\sqrt{I_1} + \sqrt{I_2})^2$	$I_{\text{min}} = (\sqrt{I_1} - \sqrt{I_2})^2$
$I_{\text{max}} = (a_1 + a_2)^2$	$I_{\text{min}} = (a_1 - a_2)^2$
If,	If,
$I_1 = I_2 = I$	$I_1 = I_2 = I$
$I_{\text{max}} = 4I$	$I_{\text{min}} = 0$

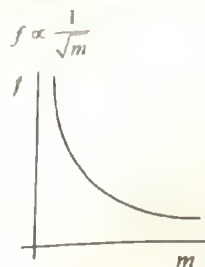
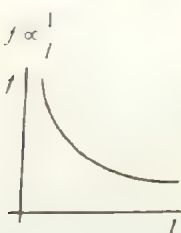
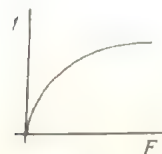
Stationary waves:

	Δx	$\Delta \phi$	Δv
1) N-N	$\frac{\lambda}{2}$	π	$\frac{1}{2}$
2) A-A	$\frac{\lambda}{2}$	π	$\frac{1}{2}$
3) N-A	$\frac{\lambda}{4}$	$\frac{\pi}{2}$	$\frac{1}{4}$

Stationary waves in a stretched string/fundamental frequency and harmonics:

$$v = \frac{1}{2l} \sqrt{\frac{F}{m}} \text{ Where } m = \frac{\text{mass}}{\text{length}}$$

$$f \propto \sqrt{F}$$



Stationary waves in stretched string or organ pipe open from both ends:

First mode	Second mode	Third mode	n th mode
1 st Harmonics Or Fundamental	2 nd Harmonics Or 1 st Overtone	3 rd Harmonics Or 2 nd Overtone	n th Harmonics Or (n-1) th Overtone
$l = \frac{\lambda_1}{2}$	$l = \lambda_2$	$l = \frac{3}{2} \lambda_3$	$l = n \frac{\lambda_n}{2}$
$\lambda_1 = 2l$	$\lambda_2 = l$	$\lambda_3 = \frac{2}{3} l$	$\lambda_n = \frac{2l}{n}$
$f_1 = \frac{v}{2l}$	$f_2 = \frac{v}{l} = 2f_1$	$f_3 = \frac{3}{2} \frac{v}{l} = 3f_1$	$f_n = n f_1$ $n = 1, 2, 3, \dots$
2N, 1A	3N, 2A	4N, 3A	No. of Nodes > A

- No of nodes = no. of loops + 1
- No of antinodes = no. of loops

Organ pipe closed from one end:

First mode	Second mode	3 rd mode	n th mode
1 st Harmonics Or Fundamental	2 nd harmonics Or 1 st Over tone	3 rd Harmonics Or 2 nd Over tone	n th Harmonics Or (n-1) st Over tone
$l = \frac{\lambda_1}{4}$	$l = \frac{3\lambda_2}{4}$	$l = \frac{5}{4} \lambda_3$	$l = n \frac{\lambda_n}{4}$ $n = 1, 3, \dots$
$\lambda_1 = 4l$	$\lambda_2 = \frac{3}{4} l$	$\lambda_3 = \frac{4}{5} l$	$\lambda_n = \frac{4l}{n}$ $n = 1, 3, \dots$
$f_1 = \frac{v}{4l}$	$f_2 = 3 \frac{v}{4l} = 3f_1$	$f_3 = 5 \frac{v}{4l} = 5f_1$	$f_n = n f_1$ $n = 1, 3, \dots$
N, A	2N, 2A	3N, 3A	Nodes = Anti nodes

$$f_{\text{open}} = 2 f_{\text{closed}}$$

$$f_{\text{closed}} = \frac{f_{\text{open}}}{2}$$

Doppler's effect:

General formula:

$$f' = \left(\frac{v \pm u_o}{v \pm u_s} \right) f \text{ or } \lambda' = \left(\frac{v \pm u_s}{v \pm u_o} \right) \lambda$$

- Direction of v always towards observer.
- If u_o and u_s along v, then u_o and u_s taken as negative.
- If u_o and u_s opposite to v, then u_o and u_s taken as positive.

1) When observer moves towards stationary source:

$$f' = \left(\frac{v + u_o}{v} \right) f$$

2) When observer moves away from stationary source:

$$f' = \left(\frac{v - u_o}{v} \right) f$$

3) When source moves towards stationary observer:

$$f' = \left(\frac{v}{v - u_s} \right) f$$

4) When source moves away from stationary observer:

$$f' = \left(\frac{v}{v + u_s} \right) f$$

5) When source and observer are moving towards each other:

$$f' = \left(\frac{v + u_o}{v - u_s} \right) f$$

6) When source and observer are moving away from each other:

$$f' = \left(\frac{v - u_o}{v + u_s} \right) f$$

7) When observer follows source moving in same direction:

$$f' = \left(\frac{v + u_o}{v + u_s} \right) f$$

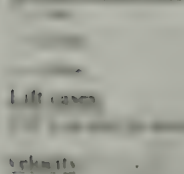
8) When source follows observer moving in same direction:

$$f' = \left(\frac{v - u_o}{v - u_s} \right) f$$

Simple Pendulum



Second Pendulum



Lift cases

Lift is moving up and with uniform

velocity

Lift is moving up and with uniform

acceleration

Lift is moving down and with uniform

acceleration

Lift is moving freely

Energy Conservation in S.H.M.

$E.T.E$
 $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$
 $\frac{1}{2}mv^2 = \frac{1}{2}k(A^2 - x^2)$
 $v = \pm \omega \sqrt{A^2 - x^2}$
 $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
 $v = \pm \omega x$

Total Energy

Total E.T.E

$\frac{1}{2}kA^2$

$\frac{1}{2}kA^2$

$\frac{1}{2}kA^2$

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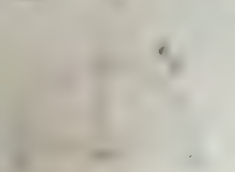
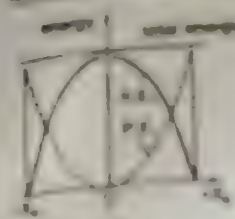
$\frac{1}{2}kA^2$

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$\frac{1}{2}kA^2$



Topic 5

Thermodynamics

Work and Heat

$W = PdV$

$W = \int PdV$

$W = \int PdV$

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$W = \int PdV$

Molar Specific Heat Capacity

$C_p = C_v + R$

$C_p = C_v + R$

$C_p = C_v + R$

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$C_p = C_v + R$

First law of thermodynamics

$\Delta U = Q - W$

$\Delta U = Q - W$

$\Delta U = Q - W$

$\Delta U = Q - W$

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Comparison of different thermodynamically process:

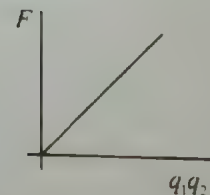
	Condition	Formula	1 st law	P-V diagram	Slope(m)	Specific heat(c)
Isothermal	$T = \text{constant}$ $\Delta T = 0$ $\Delta U = 0$	$pV = \text{constant}$	$Q = \Delta U + W$ $\Delta U = 0$ $Q = W$		Gradually Decreases	$\Delta Q = cm\Delta T$ $c = \frac{\Delta Q}{m\Delta T}$ $\Delta T = 0$ $c = \infty$
Adiabatic	$\pm Q = 0$	$pV^\gamma = \text{constant}$ $\gamma = \frac{C_p}{C_v}$	$Q = \Delta U + W$ $Q = 0$ $0 = \Delta U + W$ $-\Delta U = +W$		Rapidly decreases	$c = \frac{\Delta Q}{m\Delta T}$ $c = 0$
Isochoric	$V = \text{constant}$ $\Delta V = 0$	$\frac{P}{T} = \text{constant}$	$Q = \Delta U + W$ $Q = \Delta U + P\Delta V$ $\Delta V = 0$ $Q = \Delta U$ $\Delta U = nC_v\Delta T$		∞	$c = C_v$
Isobaric	$P = \text{constant}$ $\Delta P = 0$	$\frac{V}{T} = \text{constant}$ $V \propto T$	$Q = \Delta U + W$ $Q = \Delta U + P\Delta V$ $Q = nC_p\Delta T$		0	$c = C_p$

Topic - 6
Electrostatics

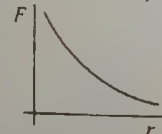
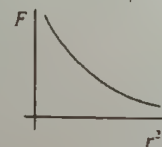
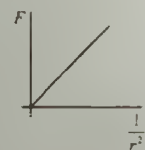
Coulomb's law:

$$F = k \frac{q_1 q_2}{r^2}$$

$$F \propto q_1 q_2$$



$$F \propto \frac{1}{r^2}$$



$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$F_{\text{vac}} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$F_{\text{med}} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \times \frac{q_1 q_2}{r^2}$$

$$\epsilon_r = \frac{F_{\text{vac}}}{F_{\text{med}}}$$

$$\epsilon_r = 1 \quad (\text{For vacuum})$$

$$\epsilon_r = 1.0006 \quad (\text{for air})$$

$$\epsilon_r > 1 \quad (\text{always})$$

• ϵ_r has no units.

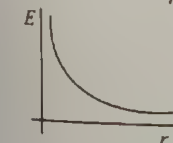
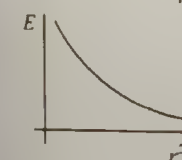
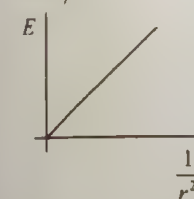
Vector Form of Coulomb's force:

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

Electric field intensity \vec{E} :

$$\vec{E} = \frac{\vec{F}}{q} = K \frac{q}{r^2} \hat{r}$$

$$E \propto \frac{1}{r^2}$$



Units:

$$E = \text{NC}^{-1} \text{ or } \text{Vm}^{-1}$$

Potential difference ΔV

$$\Delta V = \frac{\Delta W}{\Delta q} = \frac{\Delta U}{\Delta q} = \frac{K.E}{\Delta q}$$

$$\Delta W = \Delta U = K.E = q\Delta V$$

Units

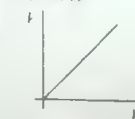
$$\text{volt} = \frac{\text{joule}}{\text{coulomb}}$$

$$V = J \text{ C}^{-1}$$

Electric potential:

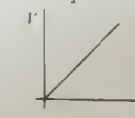
$$V = \frac{W}{q}$$

$$V \propto W$$

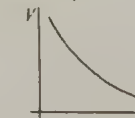


$$V = K \frac{q}{r}$$

$$V \propto q$$



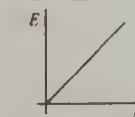
$$V \propto \frac{1}{r}$$



Electric field as potential gradient:

$$\Delta V = \frac{\Delta W}{\Delta q} = \frac{\vec{F} \cdot \vec{d}}{q}$$

$$E \propto \Delta V$$



Capacitor:

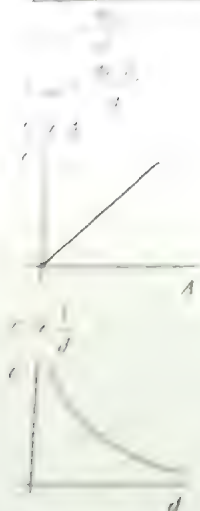
$$Q = CV$$

$$C = \frac{Q}{V}$$

$$Q \propto V$$



(capacitance of parallel plates capacitors)



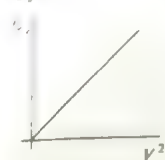
Units
coulomb
volt
farad (F)

Energy stored in a capacitor:

$U = \frac{1}{2} CV^2$
 $U = \frac{1}{2} QV$
 $U = \frac{Q^2}{2C}$



$$U \propto V$$



$$U = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} QV$$

$$U = \frac{Q^2}{2C}$$

$$\text{Energy} = \frac{1}{2} \epsilon_0 \epsilon_r E^2 (Al)$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$$

$$\text{Energy density} = \frac{1}{2} \epsilon_0 \epsilon_r \frac{V^2}{d^2}$$

Charging & discharging:

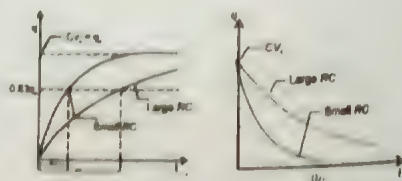
$$RC = \text{constant}$$

$$RC = \text{time}$$

$$RC = 1$$

$$\text{ohm} \times \text{farad} = \text{second}$$

$$q = q_0 (1 - e^{-t/RC})$$

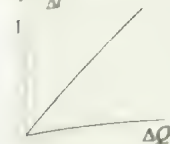


Topic - 7

Current Electricity

Electric Current:

$$I = \frac{\Delta Q}{\Delta t}$$



$$Q = ne$$

$$I = \frac{ne}{\Delta t}$$

Unit

ampere (A)

$$A = Cs^{-1}$$

Drift velocity:

$$v_d = \frac{I}{neA}$$

n = electron density

$$v_d = 10^{-3} \text{ m} \quad (\text{for metals})$$

Dimensions:

[A]

OHM's Law

$$V = IR$$

$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

$$V \propto I$$

$$V \propto R$$

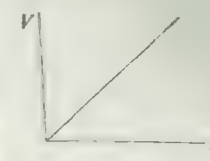
$$V \propto I$$

$$V \propto R$$

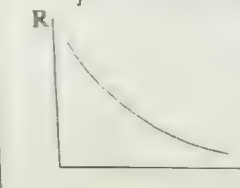
$$V \propto I$$

$$V \propto R$$

$$V \propto I$$



$$R \propto \frac{1}{I} \quad (\text{keeping } V \text{ constant})$$



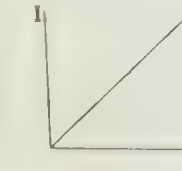
Unit of R:

Ohm (Ω)

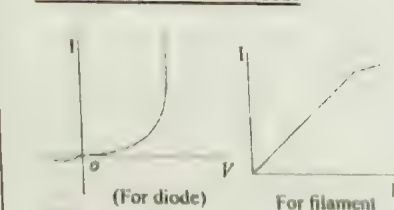
Ohmic conductors:

For metals,

$$I \propto V$$



Non Ohmic conductors:



Slopes



$$\text{Slope of } I-V \text{ Graph} = \sigma = \frac{1}{\tan \theta} = \cot \theta$$

Slope of $V-I$ Graph = $R = \tan \theta$

Specific resistance or resistivity:

$$R = \rho \frac{L}{A}$$

$$R \propto L \text{ (keeping } A \text{ constant)}$$

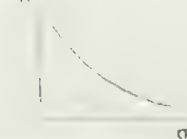
$$R \propto \frac{1}{A} \text{ (keeping } L \text{ constant)}$$

$$\rho = \frac{RA}{L}$$

Units:
 $\Omega \cdot m$

$$\text{Resistance (R)} = \frac{1}{\text{conductance}} = \frac{1}{\sigma}$$

$$R \propto \frac{1}{\sigma}$$



$$\text{conductance } = \sigma = \frac{1}{\text{resistance (R)}}$$

Units:

mho or Siemen or Ω^{-1} or \mathcal{U}

$$\text{conductivity } \propto \frac{1}{\text{Resistivity}}$$

$$\sigma = \frac{1}{\rho}$$

Units:

$\text{ohm}^{-1} \cdot \text{m}^{-1}$ or $\text{mho } \text{m}^{-1}$

Temperature coefficient of resistance

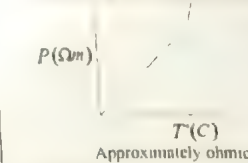
$$\alpha = \frac{R_t - R_0}{R_0 \Delta t}$$

$$\alpha = \frac{R_t - R_0}{R_0 \Delta t}$$

Units:

$$K^{-1} \text{ or } \frac{1}{K}$$

Temperature and resistivity:



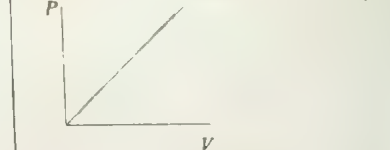
Electric power:

Work done by battery = $\Delta W = V \times \Delta Q$

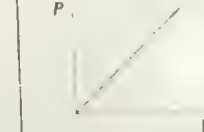
$$P = \frac{\Delta W}{\Delta t} = V \frac{\Delta Q}{\Delta t}$$

$$P = VI$$

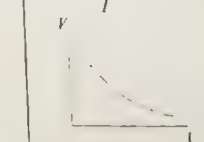
$$P \propto V \text{ (keeping } I \text{ constant)}$$



$$P \propto I \text{ (keeping } V \text{ constant)}$$



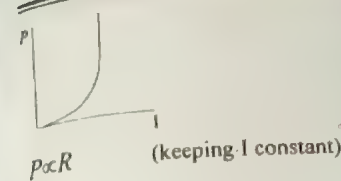
$$V \propto \frac{1}{I} \text{ (keeping } P \text{ constant)}$$



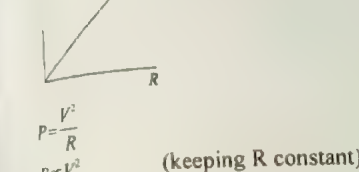
Possible formulas:

$$P = I^2 R$$

$$P \propto I^2 \text{ (keeping } R \text{ constant)}$$

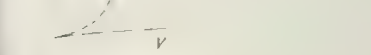


$$P \propto R \text{ (keeping } I \text{ constant)}$$



$$P = \frac{V^2}{R}$$

$$P \propto V^2 \text{ (keeping } R \text{ constant)}$$



$$P \propto \frac{1}{R} \text{ (keeping } V \text{ constant)}$$



Internal resistance of supply:

$$\text{Emf } E = \frac{\Delta W}{\Delta Q}$$

Units

volt

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

Cases:

i. Open circuit:

$$E = IR + Ir$$

$$E = IR + V_r \text{ (} I=0 \text{)}$$

ii. Discharging:

$$E = V_r + Ir$$

$$E > V_r$$

iii. Charging:

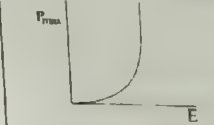
$$E = V_r + Ir$$

$$E < V_r$$

Maximum power output:

$$P_{\text{out(max)}} = \frac{E^2}{4r} = \frac{E^2}{4R}$$

$$P_{\text{max}} \propto E^2 \text{ (keeping } R \text{ constant)}$$



Brightness of bulbs:

Series combination:

$$\text{Brightness} \propto P_{\text{consumed}}$$

$$\text{Brightness} \propto I^2 R$$

$$\text{Brightness} \propto R$$

$$P_{\text{rated}} = \frac{V^2}{R}$$

$$P_{\text{rated}} \propto \frac{1}{R}$$

Parallel combination:

$$\text{Brightness} \propto P_{\text{consumed}}$$

$$\text{Brightness} \propto \frac{V^2}{R}$$

$$\text{Brightness} \propto \frac{1}{R}$$

$$P_{\text{rated}} \propto \frac{1}{R}$$

Topic - 8 Electromagnetism

Magnetic flux & flux density:

$$\phi_m = \vec{B} \cdot \vec{A}$$

$$\phi_m = BA \cos \theta$$

$$\phi_m = BA \quad (\theta = 0^\circ)$$

$$\phi_m = -BA \quad (\theta = 180^\circ)$$

$$\phi_m = \frac{\phi_{\max}}{2} = +\frac{BA}{2} \quad (\theta = 60^\circ)$$

$$\phi_m = \frac{\phi_{\max}}{\sqrt{2}} \quad (\theta = 45^\circ)$$

$$\phi_m = \frac{\sqrt{3}BA}{2} \quad (\theta = 30^\circ)$$

$$\phi_m = 0 \quad (\theta = 90^\circ)$$

$$\text{Flux density} = B = \frac{\phi_m}{A}$$

Units:

$$B = \frac{\text{weber}}{\text{meter}^2} = \text{Wb m}^{-2}$$

$$B = \text{NA}^{-1} \text{m}^{-1} = 1 \text{ T}$$

Force acting on a charge particle in a uniform magnetic field:

$$\vec{F} = q(\vec{v} \times \vec{B}) = \pm e(\vec{v} \times \vec{B})$$

$$F = qvB \sin \theta$$

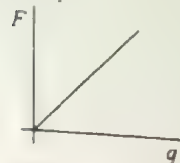
$$B = \frac{F}{qv \sin \theta}$$

$$F_{\max} = qvB \quad (\theta = 90^\circ)$$

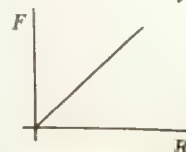
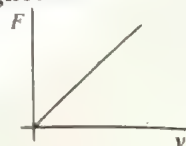
$$F_{\min} = 0 \quad (\theta = 0^\circ \text{ or } 180^\circ)$$

$$F = \frac{F_{\max}}{2} \quad (\theta = 30^\circ)$$

$$F \propto q$$



$$F \propto v$$



Path followed by charge particle in magnetic field:

If angle between \vec{v} and \vec{B} .

$\theta = 0^\circ / 180^\circ$ path is straight line.

$\theta = 90^\circ$ path is circle.

$90^\circ < \theta < 180^\circ$ path is helical.

$180^\circ < \theta < 270^\circ$ path is helical.

$$F \propto B$$

Charged particle in electric and magnetic field:

$$\vec{a} = \frac{e}{m} \vec{E}$$

Lorentz force

$$\vec{F} = \vec{F}_E + \vec{F}_B$$

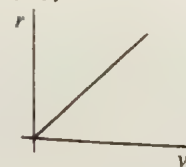
$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

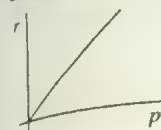
Charge to mass ratio $\frac{e}{m}$:

$$r = \frac{mv}{qB} = \frac{P}{qB}$$

$$r \propto v$$



$$r \propto p$$



$$r = \frac{q}{m} B$$

$$f = \frac{qB}{2\pi m}$$

$$T = \frac{2\pi m}{qB}$$

$$v = \frac{E}{B}$$

$$\frac{e}{m} = \frac{2v}{B^2 r^2} = \frac{E}{B^2 r}$$

$$\left(\frac{e}{m}\right)_{\text{electron}} = 1.75 \times 10^{-11} \text{ C kg}^{-1}$$

$$\left(\frac{e}{m}\right)_{\text{proton}} = 1.0 \times 10^{-8} \text{ C kg}^{-1}$$

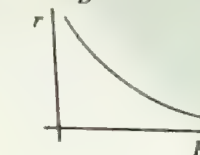
$$\left(\frac{e}{m}\right)_{\text{alpha}} = \frac{1}{2} \left(\frac{e}{m}\right)_{\text{proton}} = 0.5 \times 10^{-8} \text{ C kg}^{-1}$$

$$\left(\frac{e}{m}\right)_{\text{neutron}} = 0$$

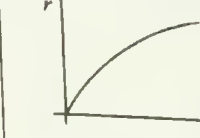
$$r = \frac{2Vm}{B^2 e}$$

$$r = \sqrt{\frac{2Vm}{B^2 e}}$$

$$r \propto \frac{1}{B} \quad (V = \text{constant})$$

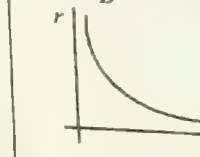


$$r \propto \sqrt{V} \quad (B = \text{constant})$$

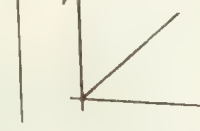


$$r = \frac{Em}{eB^2}$$

$$r \propto \frac{1}{B^2} \quad (E = \text{constant})$$



$$r \propto E \quad (B = \text{constant})$$



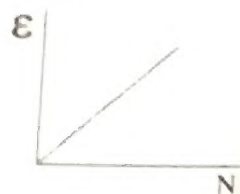
Topic - 9 Electromagnetic Induction

Faraday's Law:

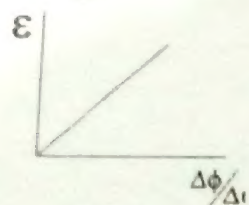
$$\mathcal{E} = -N \frac{\Delta\phi}{\Delta t}$$

$$\mathcal{E} = -\frac{NB\Delta A}{\Delta t} = \frac{NB\pi r^2}{\Delta t} \text{ (for Circular Coil)}$$

$$\mathcal{E} \propto N$$



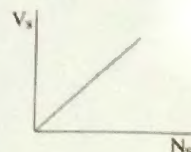
$$\mathcal{E} \propto \frac{\Delta\phi}{\Delta t}$$



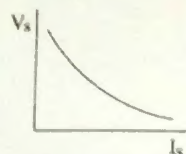
Transformer:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{I_p}{I_s}$$

$$V \propto N$$



$$V_s \propto \frac{1}{I_s}$$



Ideal Transformer:

$$P_{\text{input}} = P_{\text{output}}$$

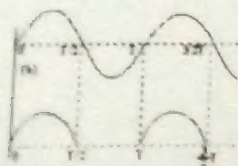
$$V_p I_p = V_s I_s$$

$$\text{Efficiency} = \eta = \frac{\text{output}}{\text{input}} = \frac{P_o}{P_i} = \frac{V_s I_s}{V_p I_p} \times 100\%$$

Topic - 10 Electronics

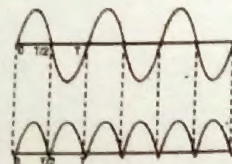
Rectification:

i. Half wave:



$$f_{\text{input}} = f_{\text{output}}$$

ii. Full wave:



$$f_{\text{input}} = 2(f_{\text{output}})$$

For filtration capacitor filter is used

Topic - 11 Dawn of Modern Physics

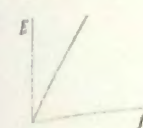
Photons:

$$E = mc^2$$

$$E = pc$$

$$E = hf$$

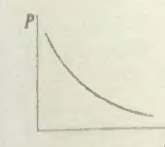
$$E \propto f$$



$$E = \frac{hc}{\lambda}$$

$$p = \frac{h}{\lambda}$$

$$p \propto \frac{1}{\lambda}$$



Wave nature of particle:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$p = \sqrt{2mE} = \sqrt{2meV}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2mK.E}}$$

$$\lambda \propto \frac{1}{\sqrt{E}} \quad (m = \text{constant})$$

$$\lambda \propto \frac{1}{\sqrt{m}} \quad (E = \text{constant})$$

$$\lambda \propto \frac{1}{\sqrt{e}} \quad (m, V \text{ are constant})$$

$$\lambda \propto \frac{1}{\sqrt{V}} \quad (\text{for same charges})$$

Determination of no. of photons

$$E_n = nhf$$

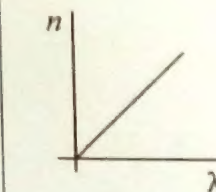
$$n = \text{number of photons}$$

$$n = \frac{E_n}{hf}$$

$$n = \frac{E\lambda}{hc} \because f = \frac{c}{\lambda}$$

$$\frac{E}{hc} = \text{Constant}$$

$$n \propto \lambda$$



Topic - 12

Atomic spectra

$$\text{Spectral Series} \rightarrow \frac{1}{\lambda} = R_H \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$$

Series	Formula	Region	Longest Wavelength	Shortest Wavelength
Lyman series	$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$ $n = 2, 3, 4, \dots, \infty$	U-V	Put $n = 2$ $\lambda = \frac{4}{3R_H}$	Put $n = \infty$ $\lambda = \frac{1}{R_H}$
Balmer series	$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$ $n = 3, 4, 5, \dots, \infty$	Visible	Put $n = 3$ $\lambda = \frac{36}{5R_H}$	Put $n = \infty$ $\lambda = \frac{4}{R_H}$
Paschen series	$\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{n^2} \right)$ $n = 4, 5, 6, \dots, \infty$	I.R	Put $n = 4$ $\lambda = \frac{144}{7R_H}$	Put $n = \infty$ $\lambda = \frac{9}{R_H}$
Bracket series	$\frac{1}{\lambda} = R_H \left(\frac{1}{4^2} - \frac{1}{n^2} \right)$ $n = 5, 6, 7, \dots, \infty$	I.R	Put $n = 5$ $\lambda = \frac{400}{9R_H}$	Put $n = \infty$ $\lambda = \frac{16}{R_H}$
Pfund series	$\frac{1}{\lambda} = R_H \left(\frac{1}{5^2} - \frac{1}{n^2} \right)$ $n = 6, 7, \dots, \infty$	I.R	Put $n = 6$ $\lambda = \frac{900}{11R_H}$	Put $n = \infty$ $\lambda = \frac{25}{R_H}$

$$R_H = \text{Rydberg's constant} = 1.0974 \times 10^7 \text{ m}^{-1}$$

$$\text{No. of transitions} = \frac{n(n-1)}{2}$$

Or

$$\text{No. of spectral series for } n^{\text{th}} \text{ state} = \frac{n(n-1)}{2}$$

$n = \text{higher energy state}$

$$\text{Ratio} = \frac{\lambda_{\text{longest}}}{\lambda_{\text{shortest}}} = \frac{n^2}{n^2 - p^2}$$

$p = \text{lower energy state}$

$$n = p + 1$$

Topic - 13

Nuclear Physics

To find no of neutron in a nucleus
 $\Rightarrow N = A - Z$

Radius of nucleus:

$$r = r_0 A^{1/3} \Rightarrow r^3 \propto A$$

$$r_0 = 1.2 \text{ fm}$$

Volume of nucleus:

$$V = \frac{4}{3} \pi r^3$$

$$V \propto r^3$$

$$V \propto A$$

Radioactivity:

α -decay	β -decay	r -decay
$X_Z^A \rightarrow X_{Z-2}^{A-4} + {}_2^4\text{He}$	${}_Z^AX \rightarrow {}_Z^{A-1}Y + {}_{-1}^0e$	${}_Z^AX \rightarrow {}_Z^AY + {}_0^0\gamma$
${}_{88}^{226}\text{Ra} \rightarrow {}_{86}^{222}\text{Rn} + {}_2^4\text{He}$	${}_{91}^{234}\text{Th} \rightarrow {}_{91}^{234}\text{Pa} + {}_{-1}^0e$	

Dependence:

Radioactivity is purely nuclear phenomenon, does not depend on temperature, pressure, electric field & magnetic field.

Half-life:

$$\Delta N = -\lambda N \Delta t$$

$$\Delta N$$

$$\lambda = \frac{N}{\Delta t}$$

Decay constant:

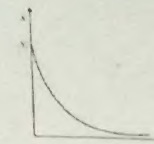
$$N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 \left(\frac{1}{2} \right)^n$$

$n = \text{no of half-lives}$

$$\frac{N}{N_0} = \frac{1}{2^n}$$

$$\Delta N \propto N$$



Graph follows this equation $N = N_0 e^{-\lambda t}$

It is exponential decay.

For decay rate (Activity)

$$A = \lambda N e^{-\lambda t}$$

Relation between λ & $T_{1/2}$

$$\lambda = \frac{0.693}{T_{1/2}}$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

This graph is for 1-128

$$\lambda T_{1/2} = 0.693$$

Mean lifetime:

$$T_{\text{mean}} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693}$$

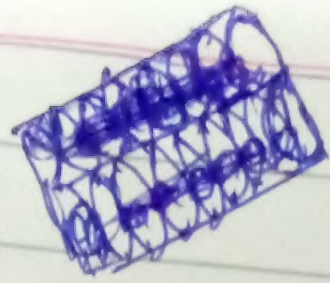
$$T_{\text{mean}} \propto T_{1/2}$$

$$T_{\text{mean}} \propto \frac{1}{\lambda}$$

For carbon dating

$$t = \frac{1}{\lambda} \ln \left(\frac{N_0}{N} \right)$$

Hidden
Expert



Regards

- Amjad Mushtaq
- Tafar Laghari
- Nabeel-ur-Rehman Baloch

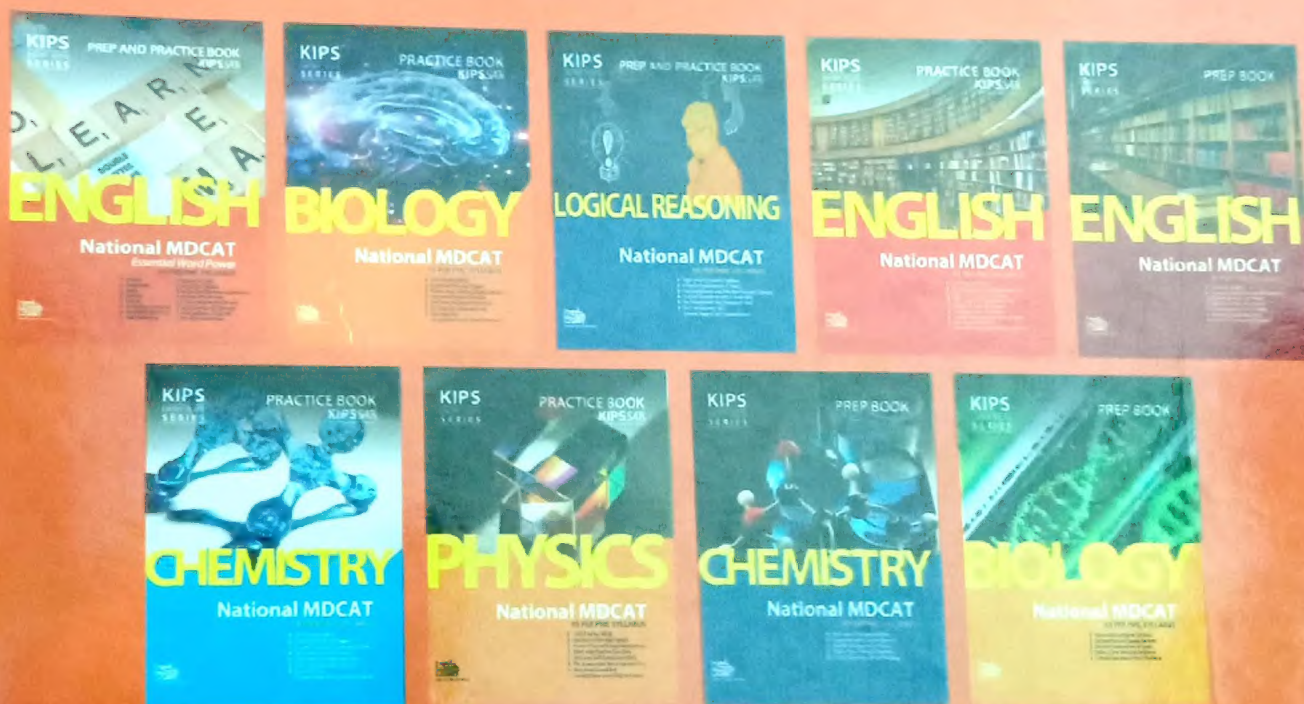
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